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445. A SUPPLEMENT TO KARANIKOLOV'S PAPER*

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Recently [**1**] CH. KARANIKOLOV has proved the following two theorems:

Theorem 1. *Let* $k \geqq \sqrt{e}$; *then the inequality*

(1) $$\pi(kx) < k\pi(x)$$

holds for every $x \geqq 347$.

Theorem 2. *Let* $0 < k \leqq e^{-1/2}$; *then the inequality*

(2) $$\pi(kx) > k\pi(x)$$

holds for all x *with* $kx \geqq 347$.

In this paper the domains of validity of the inequalities (1) and (2) will be extended.

Starting with the inequality [**2**, p. 379]

$$\frac{x}{\log x - \frac{1}{2}} < \pi(x) < \frac{x}{\log x - \frac{3}{2}} \qquad (x \geqq 67)$$

we find

$$k\pi(x) > \frac{kx}{\log x - \frac{1}{2}} \geqq \frac{kx}{\log x + (\log k - 1) - \frac{1}{2}} > \pi(kx)$$

for all $x \geqq 67$ and $\log k - 1 \geqq 0$ (i.e. $k \geqq e$).

Hence,

(3) $$\pi(kx) < k\pi(x) \qquad (k \geqq e,\ x \geqq 67).$$

On the basis of this result, we find for $67 \cdot 2^{-n} \leqq x$ and $k \geqq 2^n e$ $(n = 1, 2, \ldots)$

(4) $$\pi(kx) = \pi(k\, 2^{-n} \cdot 2^n x) < k\, 2^{-n} \pi(2^n x).$$

If $2 \leqq \min(u, v) \leqq 500$ the following inequality holds [**2**, p. 377]

(5) $$\pi(u + v) \leqq \pi(u) + \pi(v).$$

Substituting $u = v = 2^{r-1}x$ $(2 \leqq 2^{r-1}x \leqq 500)$ in (5), we have

$$\pi(2^r x) \leqq 2\pi(2^{r-1}x),$$

i.e.,

(6) $$2^{-r}\pi(2^r x) \leqq 2^{1-r}\pi(2^{r-1}x) \qquad (r = 1, \ldots, n;\ 2^{2-r} \leqq x \leqq 2^{1-r} \cdot 500).$$

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* Presented June 28, 1973 by D. S. MITRINOVIĆ.

By adding (6) we find

(7) $$2^{-n}\pi(2^n x)\leqq\pi(x) \qquad (2\leqq x\leqq 2^{1-n}\cdot 500).$$

Combining (4) and (7) we deduce

$$\pi(kx)<k\pi(x) \qquad (\max(2,\ 2^{-n}67)\leqq x\leqq 2^{1-n}500,\ k\geqq 2^n e;\ n=1,2,\ldots)$$

Therefore we have proved the following:

Theorem 3. *If* $2\leqq x<2^{-5}67$ *the inequality* (1) *holds for* $k\geqq 2^6 e$.

If $2^{-n}67\leqq x<2^{1-n}67$ $(n=1,\ldots,5)$, (1) *is valid for* $k\geqq 2^n e$.

If $67\leqq x<347$, *the inequality* (1) *holds for* $k\geqq e$.

If $x\geqq 347$, *the inequality* (1) *is valid for* $k\geqq\sqrt{e}$.

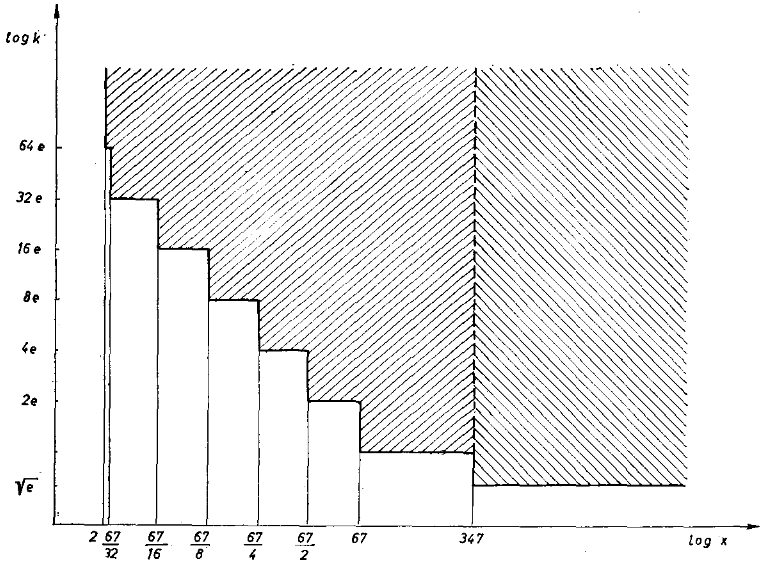

Fig. 1.

In Fig. 1 the domain of validity of (1) resulting from Theorem 3 is compared with that obtained by KARANIKOLOV's theorem.

Substituting $\frac{1}{k}$ for k and kx for x in theorem 3, we obtain:

Theorem 4. *If* $2\leqq kx<2^{-5}67$ *the inequality* (2) *holds for* $0<k\leqq 2^{-6}e^{-1}$.

If $2^{-n}67\leqq kx<2^{1-n}67$ $(n=1,\ldots. 5)$, (1) *is valid for* $0<k\leqq 2^{-n}e^{-1}$.

If $67\leqq kx<347$, *the inequality* (2) *holds for* $0<k\leqq e^{-1}$.

If $kx\geqq 347$, *the inequality* (2) *holds for* $0<k\leqq e^{-1/2}$.

REFERENCES

1. CH. KARANIKOLOV: *On some properties of function* $\pi(x)$. These Publications № **357** — № **380** (1971), 29—30.

2. D. S. MITRINOVIĆ: *Analitičke nejednakosti*. Beograd 1970.