PUBLIKACIJE ELEKTROTEHNIČKOG FAKULTETA UNIVERZITETA U BEOGRADU
PUBLICATIONS DE LA FACULTÉ D'ÉLECTROTECHNIQUE DE L'UNIVERSITÉ À BELGRADE

SERIJA: MATEMATIKA I FIZIKA — SÉRIE: MATHÉMATIQUES ET PHYSIQUE

№ 330 — № 337 (1970)

336. SOME INEQUALITIES FOR A SIMPLEX*

Miomir Stanković

Theorem 1. *Let the simplex* S_{n+1} *be determined by the points* $A_1, \ldots, A_n$, A_{n+1} *in the Euclidean n-dimensional space, and let* M *a point in its interior. Let the simplex* S_i *be determined by the points* $A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_{n+1}$. *If* T_i *is the intersection of* A_iM *with the simplex* S_i, *we have*

(1) $$\min(h_1, \ldots, h_{n+1}) \leqq \sum_{i=1}^{n+1} d_i \leqq \max(h_1, \ldots, h_{n+1}),$$

where h_i, d_i *denote the distances from* A_i *and* M *to the* $(n-1)$*-dimensional hyperplane of the simplex* S_i, *respectively.*

Proof. If V and V_i are the volumes of the simplexes S_{n+1} and M_i, which is obtained from S_{n+1}, when the point A_i is replaced by M, then

$$\frac{d_i}{h_i} = \frac{V_i}{V}$$

or

(2) $$d_i = \frac{h_i V_i}{V}.$$

Since

$$\min(h_1, \ldots, h_{n+1}) \Big(\sum_{i=1}^{n+1} V_i \Big) \leqq \sum_{i=1}^{n+1} h_i V_i \leqq \max(h_1, \ldots, h_{n+1}) \Big(\sum_{i=1}^{n+1} V_i \Big),$$

and since $\sum_{i=1}^{n+1} V_i = V$, from (2) we get (1).

Theorem 2. *Let* r_n *be the radius of the hypersphere inscribed in the simplex* S_n, *and let* r_{nk} *be the radii of the hyperspheres escribed*[1] *to that simplex. Then, we have*

(3) $$\prod_{i=1}^{n+1} r_{ni} \geqq \left(\frac{n+1}{n-1}\right)^{n+1} r_n^{\,n+1}$$

---

[1] Under a hypersphere escribed to a simplex, we understand here a hypersphere touching exactly one of the faces of the simplex from the outside.

* Presented September 1, 1970 by V. Devidé.

and

(4) $$\sum_{i=1}^{n+1} \frac{r_n}{r_{ni}-r_n} \geqq \frac{(n-1)(n+1)}{2}.$$

Proof. Since (see [1])

(5) $$\sum_{i=1}^{n+1} \frac{1}{r_{ni}} = (n-1)\frac{1}{r_n},$$

the product $\prod_{i=1}^{n+1} \frac{1}{r_{ni}}$ will have, for r_n fixed, its maximal value if

$$\frac{1}{r_1} = \cdots = \frac{1}{r_{(n+1)i}} = \frac{n-1}{(n+1)r_n}.$$

This implies that the product $\prod_{i=1}^{n+1} r_{ni}$ has the minimal value $\left(\frac{n+1}{n-1}\right)^{n+1} r_n^{n+1}$ if $r_1 = \cdots = r_{(n+1)i} = \left(\frac{n+1}{n-1}\right) r_n$.

This proves inequality (3).

From (5) it follows directly that

$$\sum_{i=1}^{n+1} \frac{r_{ni}-r_n}{r_{ni}} = 2,$$

and from

$$\left(\sum_{i=1}^{n+1} x_i\right)\left(\sum_{i=1}^{n+1} \frac{1}{x_i}\right) \geqq (n+1)^2 \quad (x_i > 0, \quad i = 1, \ldots, n+1),$$

we have

$$\sum_{i=1}^{n+1} \frac{r_{ni}}{r_{ni}-r_n} \geqq \frac{(n+1)^2}{2},$$

i.e.,

$$\sum_{i=1}^{n+1} \frac{r_n}{r_{ni}-r_n} = \sum_{i=1}^{n+1} \frac{r_{ni}-(r_{ni}-r_n)}{r_{ni}-r_n} \geqq \frac{(n+1)^2}{2} - (n+1) = \frac{(n-1)(n+1)}{2}.$$

REFERENCES

1. V. Devidé: *Poopćenje dveju teorema elementarne planimetrije na n-dimenzionalni prostor*. Glasnik mat.-fiz. astr. **4** (1951), 145-154.

2. Ž. Živanović: *Inequalities for simplex*. These Publications **№ 230** — **№ 241** (1968), 37-38.

Elektronski fakultet
Niš, Jugoslavija