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ANALITIČKA TEORIJA REZONANTNE APSORPCIJE NEUTRONA U HETEROGENIM REAKTORSKIM SISTEMIMA SA CILINDRIČNOM GEOMETRIJOM

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ANALYTICAL THEORY OF RESONANCE ABSORPTION OF NEUTRONS IN HETEROGENEOUS REACTOR SYSTEMS WITH CILYNDRICAL GEOMETRY

Kerim F. Slipičević

1. INTRODUCTION

Much work has been done on the determination of neutron resonance absorption in heterogeneous thermal reactors. The practical purpose of these attempts has been to determine the effective resonance integral, i.e. such values of the cross section that will give a correct value of the absorption rate in the resonance region assuming the shape of the neutron flux which exist in absence of the resonance. All these works can be classified in the category of analytical [**3**] and numerical procedures [**4, 5, 6**].

The basis of all the procedures are WIGNER's [**1**] or GUREVICH—POMERANCHUK's model [**2**]. Due to the complexity of the problem a considerable number of approximations is required for its solution. Depending on the type of approximation various expressions for the effective resonance integral have been derived.

In the present work this problem has been treated analytically. The use of WIGNER's rational approximation for the escape probability from the cilynder has been avoided, while for the shape of the cross section line in resonance, DOPPLER broadened BREIT—WIGNER's form was used, thus inherently introducing two main corrections of ROTHENSTEIN's procedure [3] into the expression of the effective resonance integral.

2. THE THEORY OF RESONANCE ABSORPTION OF NEUTRONS IN HETEROGENEOUS CONSTITUTION

2.1. Derivation of the Effective Resonance Integral on the Basis of Absorbed Neutron Currents

In order to derive an expression for the effective resonance integral we will consider one single fuel element of cylindrical shape. Other basic geometries may be treated by an analogous procedure. The neutrons with the energy inside the resonance will be divided into two groups, according to its last collisions:

a) Neutrons with the last collisions in the fuel element, and

b) Neutrons with the last collisions in the moderator.

The transport theory reciprocity theorem for neutron collision densities will not be used here. This means that flat flux approximation, in principle, is not required. In other words this method gives the way to abandon this assumption.

Let $Q_0^0 dE$ be the neutron source density of the first group, i.e. the number of neutrons which fall down into the energetic interval dE inside the resonance after being collided in the block and in the unit energy interval around E' above the resonance. Then the elementary neutron current from $dV = dA dz$ (Fig. 1), which does not come to the outside surface element dS of the rod because it is absorbed, is given by

$$dS\, d^2 j_s = Q_0^0\, dE\, dA\, dz\, dS \frac{r}{\rho} \cos\beta \, \frac{1 - e^{-\Sigma_t \rho}}{4\pi\rho^2} \frac{\Sigma_a}{\Sigma_t} . \tag{2.1}$$

The meaning of the geometric quantities is given in Fig. 1, and the cross section notation is usual:

Σ_t — total cross section

Σ_a — absorption cross section

Σ_{rs} — scattering cross section in the resonance

Σ_p — potential cross section per absorber atom.

If we denote with $Q_1^0\, dE$ the neutron source density in the moderator defined in a similar way as $Q_0^0 dE$ (Fig. 2), then the elementary neutron current which comes to the same element dS and will be absorbed in the rod is

$$dS\, d^2 j_m = Q_1^0\, dE\, dA\, dz\, dS \frac{r_1 \cos\beta}{\rho_1} \frac{e^{-\Sigma_1 \rho_1}}{4\pi\rho_1^2} (1 - e^{-\Sigma_t \rho_0}) \frac{\Sigma_a}{\Sigma_t} . \tag{2.2}$$

If we assume that the neutron flux is cylindrically symmetric around the rod and the rod is long enough, the source densities can be expressed as

$$Q_0^0 dE = \Phi_g(r, E')\, \Sigma_{sg}(E') \frac{dE}{(1-\alpha_0) E'} \qquad Q_1^0 dE = \Phi_m(r, E')\, \Sigma_{sm}(E') \frac{dE}{(1-\alpha_1) E'} .$$

After introducing these expressions into (2.1) and (2.2) and integrating over z, we have

$$(2.3) \qquad dj_s = \frac{dE}{E'} \frac{\Phi_g(r, E')\, \Sigma_{sg}(E')}{\Sigma_t(1-\alpha_0)}\, dA \frac{r}{4\pi} \cos\beta \int_{-\infty}^{+\infty} \frac{1-e^{-\Sigma_t \rho}}{\rho^3}\, dz$$

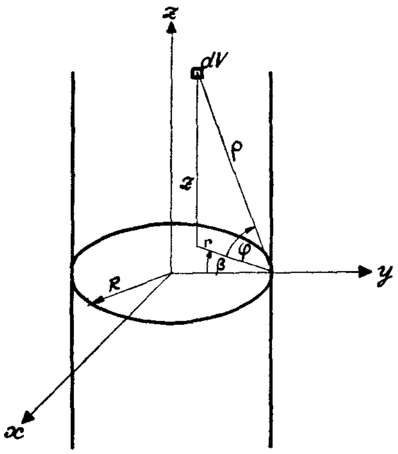

Fig. 1

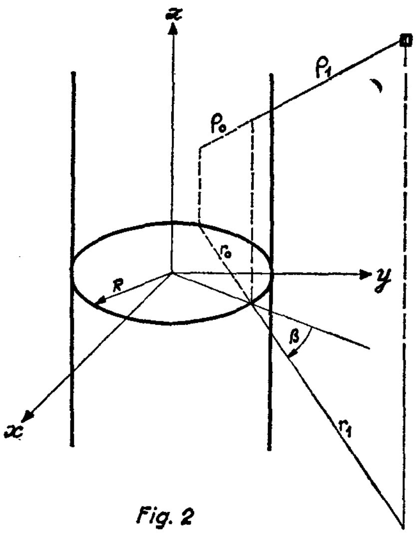

Fig. 2

$$(2.4) \qquad dj_m = \frac{dE}{E'} \frac{\Phi_m(r, E_,)\, \Sigma_{sm}(E')}{4\pi\Sigma_t(1-\alpha_1)}\, \Sigma_a dA r_1 \cos\beta \int_{-\infty}^{+\infty} \frac{e^{-\Sigma_1 \rho_1}}{\rho_1^{\,3}}\, dz$$

$$- \frac{dE}{E'} \frac{\Phi_m(r, E')\, \Sigma_{sm}(E')}{4\pi\Sigma_t(1-\alpha_1)}\, \Sigma_a dA r_1 \cos\beta \int_{-\infty}^{+\infty} \frac{e^{-(\Sigma_1\rho_1 + \Sigma_t\rho_0)}}{\rho_1^{\,3}}\, dz.$$

Integral (2.3) can be divided into two parts. From Fig. 1 it is seen that $\rho = \sqrt{r^2 + z^2}$ and the first integral iz

$$(2.5) \qquad \int_{-\infty}^{+\infty} \frac{dz}{(r^2+z^2)^{3/2}} = \frac{2}{r^2}$$

while the second

$$\int_{-\infty}^{+\infty} \frac{e^{-\Sigma_t \rho}}{\rho^3}\, dz$$

by substituting

$$(2.6) \qquad \rho = r \operatorname{ch} u \qquad\qquad z = r \operatorname{sh} u$$

comes to

$$(2.7)\qquad \frac{2}{r^2}\int_0^{\infty}\frac{e^{-\Sigma_t r\,\mathrm{ch}\,u}}{\mathrm{ch}^2 u}\,du=\frac{2}{r^2}\,Ki_2(\Sigma_t r).$$

Here, $Ki_2(\Sigma_t r)$ is BICKLEY'S function of the second order*.

Using a similar substitution

$$(2.8)\qquad \rho_1=r_1\mathrm{ch}\,u,\quad \rho_0=r_0\mathrm{ch}\,u,\quad z=r_1\mathrm{sh}\,u$$

(2.4) can be brought to this form

$$(2.9)\qquad dj_m=\frac{dE}{E'}\,\frac{\Phi_m(r,E')\,\Sigma_{sm}(E')}{4\pi\Sigma_t(1-\alpha_1)}\,\Sigma_a\,dA\cos\beta\,\frac{2}{r_1}\,\mathrm{Ki}_2(\Sigma_1 r_1)-$$

$$-\frac{dE}{E'}\,\frac{\Phi_m(r,E')\,\Sigma_{sm}(E')}{4\pi\Sigma_t(1-\alpha_1)}\,\Sigma_a\,dA\cos\beta\,\frac{2}{r_1}\,\mathrm{Ki}_2(\Sigma_1 r_1+\Sigma_t r_0).$$

In order to perform integration over the whole volume of the source, it is necessary to know the functional dependence of $\Phi_g(r,E')$ and $\Phi_m(r,E')$. The simple physical consideration of the flux problem shows that these functions do not separate the variables. Determination of the functions even in much simpler situations of purely scattering media **[25]** is very complicated. So it seems that exact treatment of this problem is practicaly impossible, but different assumptions about the functions can be made. This category includes a numerical solution of the integral equation for collision densities in **[6]**. A similar thing was done in **[10]**, where semiempirically from **[6]** correction was made for the volume term in the global value of I_{eff}, while in **[14]** the same was done for a single resonance.

Further treatment in this work is based on the flat flux assumption in both media. From (2.4) and (2.9) it is seen how other assumtions for fluxes can be introduced.

In order to perform volume integration let us write

$$(2.10)\qquad dA=r\,d\beta\,dr$$

and the total absorbed current from the volume in the fuel is found by integrating (2.3) over r and β

$$(2.11)\qquad J_s=\frac{Q_0^0\Sigma_a}{\Sigma_t}\,\frac{1}{2\pi}\left[\int_{\beta}\cos\beta\,d\beta\int_0^{2R\cos\beta}dr-\int_{\beta}\cos\beta\,d\beta\int_0^{2R\cos\beta}Ki_2(\Sigma_t r)\,dr\right]\frac{dE}{(1-\alpha_0)\,E'}.$$

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* In general, BICKLEY'S function of order n **[25]** is defined by

$$Ki_n(x)=\int_0^{\infty}\frac{e^{-x\,\mathrm{ch}\,t}}{\mathrm{ch}^n t}\,dt$$

Some properties of this function used in this work are given in the text.

Using the following results

$$(2.12)\qquad \int_x^{\infty} Ki_n(x)\,dx = Ki_{n+1}(x) \qquad Ki_3(0) = \frac{\pi}{4} \qquad \int_{-\pi/2}^{+\pi/2} \cos^2 x\,dx = \frac{\pi}{2}$$

and an alternative way of writing

$$(2.13)\qquad \int_0^{2R\cos\beta} Ki_2(\Sigma_t r)\,dr = \int_0^{\infty} Ki_2(\Sigma_t r)\,dr - \int_{2R\cos\beta}^{\infty} Ki_2(\Sigma_t r)\,dr$$

for the total absorbed current per unit outside surface from source in the fuel, we have

$$(2.14)\qquad J_s = \left[\frac{Q_0^0}{2}\frac{\Sigma_a}{\Sigma_t}R - \frac{Q_0^0}{4}\frac{\Sigma_a}{\Sigma_t^2} + \frac{Q_0^0}{2\pi}\frac{\Sigma_a}{\Sigma_t^2}\int_{-1}^{+1} Ki_3(2R\Sigma_t\sqrt{1-u^2})\,du\right]\cdot\frac{dE}{(1-\alpha_0)E'}$$

where $u = \sin\beta$.

In a similar way by integration of (2.9) over the whole moderator volume, where it is now

$$(2.15)\qquad dA = r_1 d\beta\, dr_1$$

and since for a given β, r is fixed, the total absorbed current in the block per its unit surface, which comes from the source in the moderator, is expressed by

$$(2.16)\qquad J_m = \left[\frac{Q_1^0\Sigma_a}{4\Sigma_1\Sigma_t} - \frac{Q_1^0\Sigma_a}{2\pi\Sigma_t\Sigma_1}\int_{-1}^{+1} Ki_3(2R\Sigma_t\sqrt{1-u^2})\,du\right]\frac{dE}{(1-\alpha_1)E'}.$$

For the normalized flux of one neutron per cell we write

$$(2.17)\qquad \Phi = \frac{1}{\overline{V\xi\Sigma_s}E'}$$

where $\overline{V\xi\Sigma_s} = V_0\xi_0\Sigma_{po} + V_1\xi_1\Sigma_{s1}$.

Indexes 0 and 1 denote the fuel and moderator, respectively.

The effective resonance integral is such value of the cross section which in the expression for the absorption rate, with a flux which exists in the absence of the resonance, gives a correct value of the absorption rate

$$(2.18)\qquad R = N_0 I_{eff}\Phi V_0$$

where N_0 is the atomic absorber density and V_0 is the volume of the rod.

The absorption rate is already determined by (2.14) and (2.16) and by equating we obtain

$$(2.19)\qquad S_0\int_{E_0-\Delta E}^{E_0+\Delta E} dE\left\{\int_E^{E/\alpha_0} J_s\frac{dE'}{(1-\alpha_0)E'} + \int_E^{E/\alpha_1} J_m\frac{dE'}{(1-\alpha_1)E'}\right\} = N_0 I_{eff}\Phi V_0.$$

Introducing (2.14) and (2.16) into (2.19) and by suitable grouping of the terms, we write

(2.20) $$I_{eff} = \frac{1}{N_0} \int_{E_0-\Delta E}^{E_0+\Delta E} \frac{\Sigma_p \Sigma_a}{\Sigma_t} \frac{dE}{E} + \frac{S_0}{4 N_0 V_0} \int_{E_0-\Delta E}^{E_0+\Delta E} \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t^2} G \frac{dE}{E}$$

where

(2.21) $$G = 1 - \frac{4}{\pi} \int_0^1 Ki_3 (2 R \Sigma_t \sqrt{1-u^2}) du.$$

As is seen, the accepted assumption for the fluxes brought us to the known expression for the effective resonance integral. An expression similar to (2.21) for the penetrability factor G was derived by ROTHENSTEIN [26] but in [3] it is not given. Attempts were not made to get a suitable approximation for the penetrability factor G on the basis of (2.21).

2.2. New Approximate Expression for the Penetrability Factor G.

TAYLOR series expansion arround $2 R \Sigma_t$ of the function G gives the possibility to write instead of (2.21) the following

(2.22) $$G = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{Ki_3^{(n)} (2R \Sigma_t)}{n!} (2 R \Sigma_t)^n C_n$$

where

(2.23) $$C_n = \int_0^1 (\sqrt{1-u^2} - 1)^n \, du.$$

The solution to this integral is

$$C_n = \frac{(-1)^n}{n+1} \left\{ 2 + \frac{2n-1}{n-1} + \frac{(2n-1)(2n-3)}{(n-1)(n-2)} + \cdots - \frac{(2n-1)(2n-3)\cdots[2n-(2n-1)]}{(n-1)(n-2)\cdots[n-(n-1)]} \frac{\pi}{2} \right\}$$

The first ten values for this coefficient are:

$C_0 = 1.00000000000$	$C_3 = -0.05475688725$	$C_6 = -0.0185383130$
$C_1 = -0.21460183660$	$C_4 = 0.03554618953$	$C_7 = -0.0143122181$
$C_2 = 0.09587033986$	$C_5 = -0.02498243870$	$C_8 = 0.0113883522$
		$C_9 = -0.0092802468$

The numerical examination of (2.22) is shown in T-1 and Fig. 3. The three-term series gives the maximal error in the whole range of 2.5%, while the five-term series gives the same error of 1.1%. In Fig. 3 besides the two- and three-term series for G, the WIGNER rational approximation is also given for comparison.

The further advantage of the expression (2.22) is in the fact that the series with any number of terms gives correct limiting values:

a) for $R \to 0$ $G \to 0$.

TABLE I

x	3 terms G	Δ (%)	5 terms G	Δ (%)
0.	0.0	0.0	0.0	0.0
0.1	0.1771849834	0.1022	0.1770243687	0.0023
0.2	0.3181723923	0.3026	0.3172380514	0.0082
0.3	0.4322473319	0.5473	0.4299773103	0.0193
0.4	0.5255314569	0.8124	0.5215021268	0.0395
0.5	0.6023652938	1.0764	0.5963376446	0.0649
0.6	0.6660410259	1.3298	0.6579457791	0.0981
0.8	0.7628560556	1.7163	0.7509357064	0.1269
1.0	0.8315765719	2.1216	0.8167680667	0.3031
1.5	0.9275109060	2.5202	0.9104768039	0.6374
2.0	0.9687299269	2.421	0.9562866705	1.1088
3.0	0.9943391856	1.758	0.9879318737	1.1024
4.0	0.9990081474	1.1469	0.9969360383	0.9371
5.0	0.9998308586	0.7589	0.9992732155	0.7027

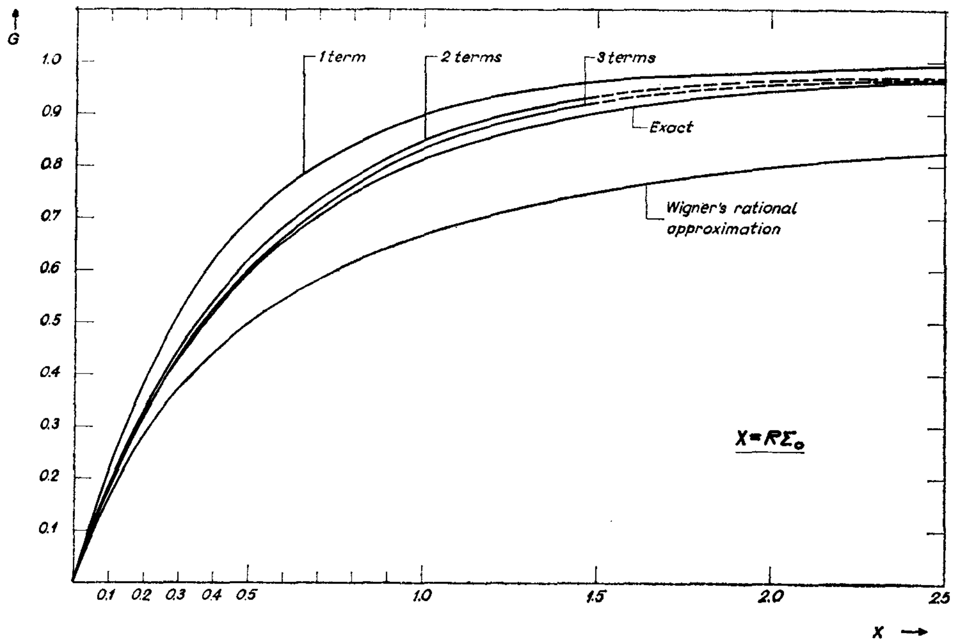


Fig. 3

The first term goes to 1, while the second and the third go to zero, since $Ki_2(0)=1$ and $Ki_1(0)=\frac{\pi}{2}$. Higher derivatives than three of the function $Ki_3(x)$ gives modified Bessel's functions of the second kind, and these functions have the singularity in the origin of the coordinate system. But for small x

$$K_0(x) \sim -\log x \tag{2.24}$$

and $K_n(x) \sim 2^{n-1}(n-1)!\, x^{-n}$ and the limiting values are correct

$$\lim_{x\to 0} x^3 K_0(x) \to 0 \quad \text{and} \quad \lim_{x\to 0} x^{n+3} K_n(x) \to 0.$$

b) For $R \to \infty$ $G \to 1$.

Asymptotic expansions of the functions $Ki_n(x)$ and $K_n(x)$ are given by

$$e^{-x}\sqrt{\frac{\pi}{2x}}$$

so that

$$\lim_{x\to\infty} x^n e^{-x}\sqrt{\frac{\pi}{2x}} \to 0$$

which means that every term of the series separately goes to zero and G goes to 1.

2.3. New Approximate Expression for the Effective Resonance Integral on the Basis of the Approximate Expression for the Penetrability Factor

We can write (2.22) in a more developed form

$$G = 1 - \frac{4}{\pi}\Big\{ |C_0|\, Ki_3(2R\Sigma_t) + |C_1|\, Ki_2(2R\Sigma_t)\cdot 2R\Sigma_t + \frac{|C_2|}{2} Ki_1(2R\Sigma_t)(2R\Sigma_t)^2 + \sum_{n=3}^{\infty} |C_n| \frac{Ki_3^{(n)}(2R\Sigma_t)}{n!}(2R\Sigma_t)^n \Big\}. \tag{2.25}$$

Using such a procedure we extracted from the penetrability factor G those terms which will, from the surface term of the effective resonance integral, extract the terms explicitely non dependent on the ratio S_0/V_0, or dependent on the first power. After expanding each of these terms around $q2R\Sigma_0$, where q is a parameter between 0 and 1, we can write

$$G = 1 - \frac{4}{\pi}\Big\{ |C_0| \sum_{n=0}^{\infty} \frac{Ki_3^{(n)}(q2R\Sigma_0)}{n!}(2R\Sigma_t - q2R\Sigma_0)^n + |C_1|(2R\Sigma_t)\sum_{n=0}^{\infty}\frac{Ki_2^{(n)}(q2R\Sigma_0)}{n!}(2R\Sigma_t - q2R\Sigma_0)^n + \frac{|C_2|}{2}(2R\Sigma_t)^2 \sum_{n=0}^{\infty}\frac{Ki_1^{(n)}(q2R\Sigma_0)}{n!}(2R\Sigma_t - q2R\Sigma_0)^n + \sum_{n=3}^{\infty}|C_n|\frac{Ki_3^{(n)}(2R\Sigma_t)}{n!}(2R\Sigma_t)^n \Big\}. \tag{2.26}$$

Introduction of this parameter q and its later determination in dependance on the parameters of the resonance, it is possible to make (2.32) equally valid for every resonance. At the same time (2.32) will have correct limiting values. This procedure is in a certain sense similar to the procedure in [3] and [10], where WIGNER's rational approximation is corrected. The difference is in the fact that they use a constant instead of the parameter.

The first three sums can be rewritten as:

For the first

$$Ki_3(q\,2\,R\,\Sigma_0)-Ki_2(q\,2\,R\,\Sigma_0)(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)+$$

$$+\sum_{n=2}^{\infty}\frac{Ki_3^{(n)}(q\,2\,R\,\Sigma_0)}{n!}(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)^n$$

and for the second

$$Ki_2(q\,2\,R\,\Sigma_0)(2\,R\,\Sigma_t)+\sum_{n=1}^{\infty}\frac{Ki_2^{(n)}(q\,2\,R\,\Sigma_0)}{n!}(2\,R\,\Sigma_t)(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)^n$$

while the third one will be kept in the same form.

Since $Ki_3^{(n+2)}=-Ki_2^{(n+1)}=Ki_1^{(n)}$ and $Ki_2^{(n+1)}=-Ki_1^{(n)}$ it is possible to combine the first three sums in (2.26) so that as a suitable expression for the penetrability factor G for introduction into the resonance integral, we write

$$(2.27)\quad G=1-\frac{4}{\pi}|C_0|Ki_3(q\,2\,R\,\Sigma_0)+$$

$$+\frac{4}{\pi}\{|C_0|Ki_2(q2\,R\,\Sigma_0)(2\,R\,\Sigma_t-q\,2\,R\Sigma_0)-|C_1|Ki_2(q\,2\,R\,\Sigma_0)(2R\,\Sigma_t)\}-$$

$$-\frac{4}{\pi}\sum_{n=0}^{\infty}(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)^n\frac{Ki_1^{(n)}(q\,2\,R\,\Sigma_0)}{n!}\Bigg[\frac{|C_0|}{(n+1)(n+2)}(2\,R\,\Sigma_t-$$

$$-q\,2\,R\,\Sigma_0)^2-\frac{|C_1|}{n+1}(2\,R\,\Sigma_t)(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)+\frac{|C_2|}{2}(2\,R\,\Sigma_t)^2\Bigg]-$$

$$-\frac{4}{\pi}\sum_{n=3}^{\infty}|C_n|\frac{Ki_3{}^{(n)}(2R\Sigma_t)}{n!}(2R\,\Sigma_t)^n.$$

After introduction of this expression for G into (2.20) for the effective resonance integral we write

$$(2.28)\quad I_{eff}=\frac{1}{N_0}\int_E\frac{\Sigma_p\Sigma_a}{\Sigma_t}\frac{dE}{E}+\frac{S_0}{V_0}\frac{1}{4\,N_0}\left[1-\frac{4}{\pi}Ki_3(q2\,R\,\Sigma_0)\right]\int_E\frac{\Sigma_a(\Sigma_t-\Sigma_p)}{\Sigma_t^2}\frac{dE}{E}+$$

$$+\frac{4}{\pi}Ki_2(q\,2\,R\,\Sigma_0)\frac{S_0}{4\,V_0N_0}\int_E\frac{\Sigma_a(\Sigma_t-\Sigma_p)}{\Sigma_t^2}\Bigg[|C_0|(2\,R\,\Sigma_t-q\,2\,R\,\Sigma_0)-$$

$$-|C_1|\,2\,R\,\Sigma_t\Bigg]\frac{dE}{E}-\frac{4}{\pi}\frac{S_0}{4\,V_0\,N_0}\Bigg\{\sum_{n=0}^{\infty}\frac{Ki_1^{(n)}(q\,2\,R\,\Sigma_0)}{n!}\Bigg[\frac{|C_0|}{(n+1)(n+2)}\int_E(2\,R\Sigma_t-$$

$$-q\,2\,R\,\Sigma_0)^{n+2}\frac{\Sigma_a(\Sigma_t-\Sigma_p)}{\Sigma_t^2}\frac{dE}{E}-\frac{|C_1|}{n+1}\int_E\frac{\Sigma_a(\Sigma_t-\Sigma_p)}{\Sigma_t^2}(2\,R\Sigma_t)(2\,R\,\Sigma_t-$$

$$- q\, 2\, R\, \Sigma_0)^{n+1} \frac{dE}{E} + \frac{|C_2|}{2} \int\limits_E \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t^2} (2\, R\, \Sigma_t)^2 (2\, R\, \Sigma_t - q\, 2\, R\, \Sigma_0)^n \frac{dE}{E} \Bigg] +$$

$$+ \sum_{n=3}^{\infty} |C_n| \int\limits_E \frac{Ki_3^{(n)} (2\, R\, \Sigma_t)}{n!} (2\, R\, \Sigma_t)^n \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t^2} \frac{dE}{E} \Bigg\} .$$

Now, we are in the position, by suitable choice of the parameter q, to get a simple approximate expression for the effective resonance integral, which will have correct limiting values and will be a good approximation in the whole range of the rod radius. We selected it so that the third term is always zero

$$\left[|C_0| \int\limits_E \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t} - |C_0|\, q\, \Sigma_0 \int\limits_E \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t^2} \frac{dE}{E} - |C_1| \int\limits_E \frac{\Sigma_a (\Sigma_t - \Sigma_p)}{\Sigma_t} \frac{dE}{E} \right] = 0$$

which gives for q, after usual substitution of the variable E by the variable $x = (E - E_0)/(\Gamma/2)$

$$q = \frac{|C_0| - |C_1|}{|C_0|} \frac{\int\limits_0^{\infty} \frac{\Psi^2}{\Psi + \beta} dx}{\int\limits_0^{\infty} \frac{\Psi^2}{(\Psi + \beta)^2} dx} \tag{2.29}$$

and after one division of the numerator

$$q = \frac{|C_0| - |C_1|}{|C_0|} \frac{\int\limits_0^{\infty} \Psi\, dx - \beta \int\limits_0^{\infty} \frac{\Psi}{\Psi + \beta} dx}{\int\limits_0^{\infty} \frac{\Psi^2}{(\Psi + \beta)^2} dx} . \tag{2.30}$$

From earlier it is known that

$$\int\limits_{-\infty}^{\infty} \Psi(\xi, x)\, dx = \pi$$

and since $\Psi(\xi, x)$ is even in x, it is

$$\int\limits_0^{\infty} \Psi(\xi, x)\, dx = \frac{\pi}{2}$$

The integrals

$$\int\limits_0^{\infty} \frac{\Psi}{\Psi + \beta} dx, \qquad \int\limits_0^{\infty} \frac{\Psi}{(\Psi + \beta)^2} dx \tag{2.31}$$

are from before known functions $J(\xi, \beta)$ and $K(\xi, \beta)$, and ultimately we write

$$q = \frac{|C_0| - |C_1|}{|C_0|} \; \frac{\pi/2 - \beta J(\xi, \beta)}{K(\xi, \beta)}. \tag{2.30a}$$

If we consider the first three terms of the expression (2.28) as a good approximation for I_{eff}, since the reminder represents two alternative series, we can write

$$I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{S_0}{V_0} \frac{\Gamma_\gamma}{E_0} \frac{1}{4N_0} K(\xi, \beta) \left[1 - \frac{4}{\pi} |C_0| Ki_3(q\, 2R\Sigma_0)\right]. \tag{2.32}$$

2.4. Analysis of the Derived Formula for the Effective Resonance Integral

Analysis of the formula (2.32) shows:

a) $R \to \infty$. Since for the cylinder $S_0/V_0 = 2/R$, and $Ki_3(\infty) = 0$, the second term obviously goes to zero, so it is

$$I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) \tag{2.33}$$

which is the effective integral for the homogeneous mixture, as it has to be.

b) $R \to 0$. Remember the earlier determined constants

$$|C_0| = 1, \quad |C_1| = \frac{1}{2}\left(2 - \frac{\pi}{2}\right)$$

and the values $Ki_3(0) = \frac{\pi}{4}$, and $Ki_2(0) = 1$. Expansion in the Taylor series of the function $Ki_3(q2R\Sigma_0)$ in the second term around zero, we write

$$I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{2}{R} \frac{\Gamma_\gamma}{E_0} \frac{1}{4N_0} K(\xi, \beta) \left[1 - \frac{4}{\pi} |C_0| (Ki_3(0) - Ki_2(0) q\, 2R\Sigma_0 + \cdots)\right].$$

Higher terms of the expansion go to zero for the earlier reasons (page 8), and since

$$\frac{|C_0| - |C_1|}{|C_0|} = \frac{\pi}{4}$$

we write, using (2.31) for q

$$I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{4}{\pi} \frac{\Gamma_\gamma}{E_0} \frac{\Sigma_0}{N_0} K(\xi, \beta) \frac{\pi}{4} \frac{\frac{\pi}{2} - \beta J(\xi, \beta)}{K(\xi, \beta)}$$

and since $\beta \sigma_0 = \sigma_p$

$$I_{eff} = \frac{\Gamma_\gamma}{E_0} \sigma_0 \frac{\pi}{2} = I_\infty \tag{2.34}$$

which is the infinite dilution limit, as it has to be.

c) As it is known, WIGNER's expression for I_{eff} is good for large radii. In the formula (2.32) for large values of the argument, Ki_3 function can be neglected, and directly comes

$$(2.35) \qquad I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{S_0}{V_0} \frac{\Gamma_\gamma}{E_0} \frac{1}{4 N_0} K(\xi, \beta).$$

d) For the case of small values of R, the argument of the function Ki_3, $x = q 2 R \Sigma_0$, because of the large value of Σ_0, can still be considered large anough so that the assymptotic expansion of the function Ki_3 [24] gives a good approximation,

$$Ki_3(x) \sim e^{-x} \sqrt{\frac{\pi}{2x}} \left[1 - \frac{B_3}{x} + \frac{C_3}{x^2} - \ldots \right].$$

Taking further $e^{-x} \sim 1 - x$, which is rather rough for the values of x for which the assymptotic expansion is valid, we easily get

$$(2.36) \qquad I_{eff} \sim \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{\Gamma_\gamma}{E_0} \frac{1}{4 N_0} K(\xi, \beta) \frac{S_0}{V_0} -$$

$$- \frac{\Gamma_\gamma}{E_0} \frac{1}{4 N_0} K(\xi, \beta) \sqrt{\frac{8}{\pi}} (1 + B_3) \frac{1}{\sqrt{4 q \Sigma_0}} \left(\frac{S_0}{V_0} \right)^{3/2}$$

$$+ \frac{\Gamma_\gamma}{E_0} \frac{1}{4 N_0} K(\xi, \beta) \sqrt{\frac{8}{\pi}} \sqrt{4 q \Sigma_0} \sqrt{\frac{S_0}{V_0}}.$$

Comparison of the last term of this formula with the formula given in [2] shows that all parameters of the resonance, dimensions of the rod and the density, appear in both of them in the same way. In our formula DOPPLER's broadened cross sections are used, while GUREVICH and POMERANCHUK get a formula with the natural line shape of the cross section curves. Furthermore, we have only in the third term of (2.36) in the numerator $\sqrt{\Sigma_0}$, and in most cases this term could be expected to be dominant. Although the derivation of the GUREVICH—POMERANCHUK formula is not as obvious as was the derivation in (c), it is possible to see that (2.32) included it.

e) The formula (2.32) gives the possibility for a suitable physical analysis of I_{eff}. For the given material constitution, the rod radius and the temperature, the following assymptotic straight line is determined

$$(2.37) \qquad I_{eff} = \frac{\sigma_p \Gamma_\gamma}{E_0} J(\xi, \beta) + \frac{\Gamma_\gamma}{E_0} \frac{1}{4 N_0} K(\xi, \beta) \frac{S_0}{V_0}.$$

The deviations of I_{eff} from this line can be discussed in terms of $Ki_3(x)$ function, which is a well-behaved function.

For the given resonance (given σ_0) and atomic absorber density, the deviation from the straight line will be larger as R is smaller. This fact is known experimentally and the Russian theory is more valid for smaller dimensions of the block.

For the materials with smaller absorber atomic density N_0 the deviations from the straight line are larger meaning that oxide and carbide fuels have »better« $\sqrt{\frac{S_0}{V_0}}$ dependence than metalic fuels, which is also experimentally known.

It seems that on the basis of this formula, analysis of the temperature dependence of I_{eff} is more simple than by other formulas, since the changes of I_{eff} with temperature can be considered in terms of the changes of the functions $J(\xi, \beta)$, $K(\xi, \beta)$ and $q(\xi, \beta)$.

f) Comparison with the *AHN* method shows that the volume terms are equal in both formulas while the surface terms are different. But it is easy to show that for the large R the function $L(t, \xi, \beta)$ in **[4]** go to

$$L \to \frac{1}{2R\Sigma_0} K(\xi, \beta).$$

In formula (2.32) for large R the function Ki_3 can be neglected. This means that both formulas for large R go to WIGNER's expression.

g) By redefinition of the parameters according to the model of the rectangular resonance **[14]**, the formula (2.32) can be brought to an intermediary type which as the limiting cases gives a narrow resonance or a narrow resonance infinite mass approximation. If the resonance is not narrow enough for the neutron to leave it in one collision, only part of the scattering cross section is effective. If p_0 is the probability that the scattered neutron in the resonance leaves it, then the effective fraction of the scattering cross section is given by $p_0\,\sigma_s$. By inverting the energy scale it is seen that the fraction of the scattering cross section outside the resonance which supplies the neutrons in the group inside the resonance is given by $p_0\,\sigma_p$.

In the case of the rectangular resonance the probability p_0 can be easily determined. If $(1-\alpha)E_0 \geqslant \Gamma_p$

$$p_0 = \frac{1}{\Gamma_p}\int_0^{\Gamma_p} \frac{[(1-\alpha)E_0 - \Gamma_p + x]}{(1-\alpha)E_0}\,dx = 1 - \frac{1}{2}\frac{\Gamma_p}{(1-\alpha)E_0} \tag{2.38}$$

and if $\Gamma_p > (1-\alpha)E_0$

$$p_0 = \frac{1}{\Gamma_p}\int_0^{(1-\alpha)E_0} \frac{x}{(1-\alpha)E_0}\,dx = \frac{1}{2}\frac{(1-\alpha)E_0}{\Gamma_p}\,. \tag{2.39}$$

After introduction of the effective values for the cross sections

$$\sigma_p^* = p_0\,\sigma_{p0} + p_m\,\sigma_m \qquad \sigma_t^* = \sigma_a + p_0\,\sigma_{rs} + p_0\,\sigma_{p0} + p_m\,\sigma_m \tag{2.40}$$

the formula (2.32) can be written

$$I_{eff} = \frac{\Gamma_\gamma\,\sigma_p^*}{E_0 f(p_0)} J(\xi, \beta^*) + \frac{S_0}{V_0}\frac{1}{4N_0}\frac{\Gamma_\gamma}{E_0} K(\xi, \beta^*)\left[1 - \frac{4}{\pi}|C_0|\,Ki_3\left(q^* 2R\Sigma_0^*\right)\right]. \tag{2.41}$$

h) As in the *AHN* method in case of a wide resonance in a pure metallic fuel, the surface term requires a separate formulation. Although, in principle, the procedure of Section 2.1. can be used, this was not done since the formulation of *AHN* gives this term as a function of two independent variables and therefore nothing more could be obtained.

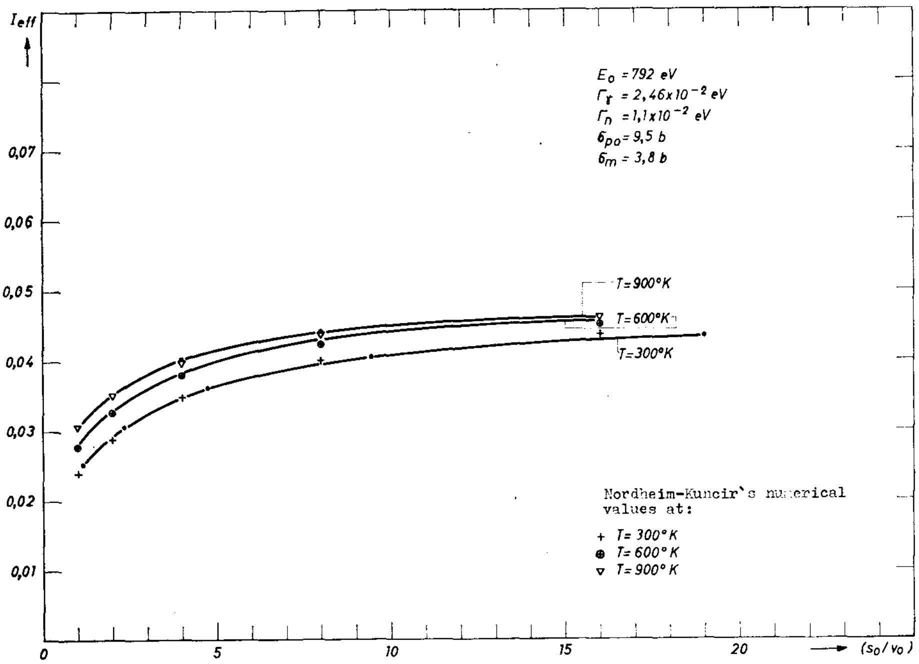

Ieff
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
5
10
15
20
(so/vo)
Eo = 792 eV
Γγ = 2,46x10⁻² eV
Γn = 1,1x10⁻² eV
σpo = 9,5 b
σm = 3,8 b
T=900°K
T=600°K
T=300°K
Nordheim-Kuncir's numerical values at:
+ T= 300° K
⊕ T= 600° K
▽ T= 900° K

FIG. 4

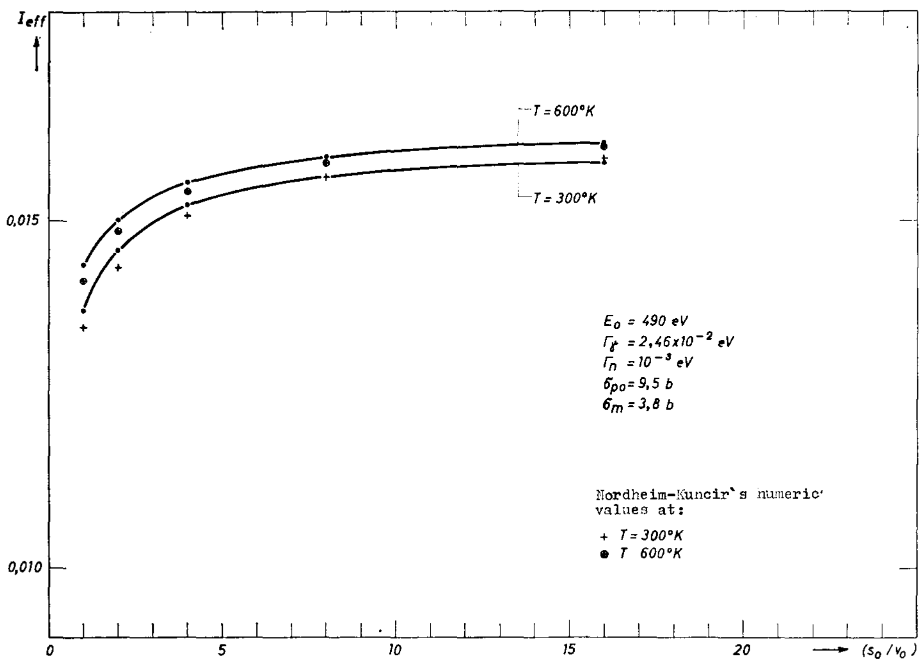

Ieff
0,015
0,010
0
5
10
15
20
(so / vo)
T = 600°K
T = 300°K
Eo = 490 eV
Γγ = 2,46x10⁻² eV
Γn = 10⁻³ eV
σpo = 9,5 b
σm = 3,8 b
Nordheim-Kuncir's numeric values at:
+ T = 300°K
● T 600°K

FIG. 5

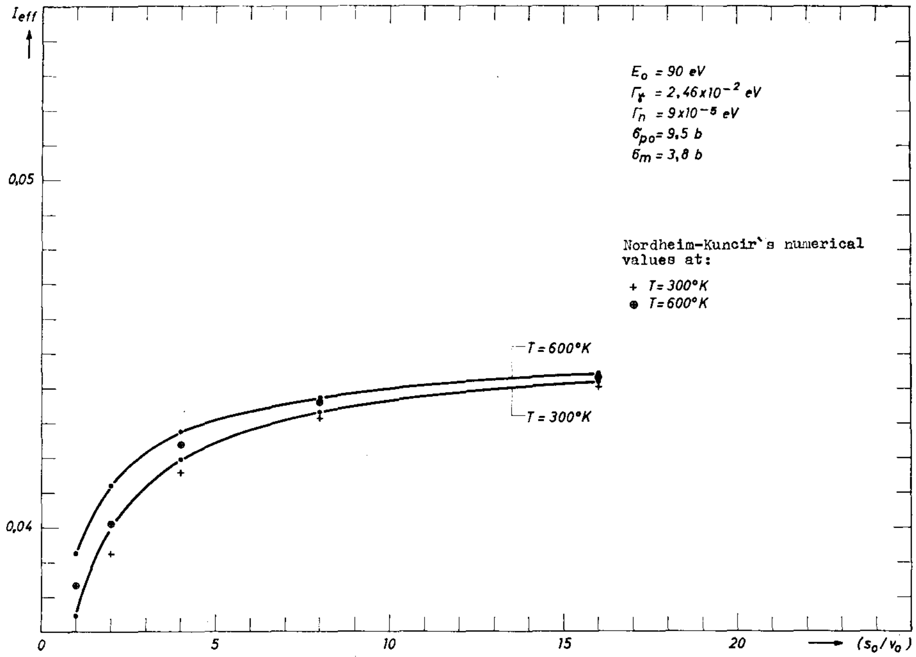

I_{eff}
0,05
0,04
E_o = 90 eV
Γ_γ = 2,46x10^{-2} eV
Γ_n = 9x10^{-5} eV
σ_{po}= 9,5 b
σ_m = 3,8 b
Nordheim-Kuncir's numerical values at:
+ T=300°K
● T=600°K
T = 600°K
T = 300°K
0
5
10
15
20
(s_o/v_o)

FIG.6

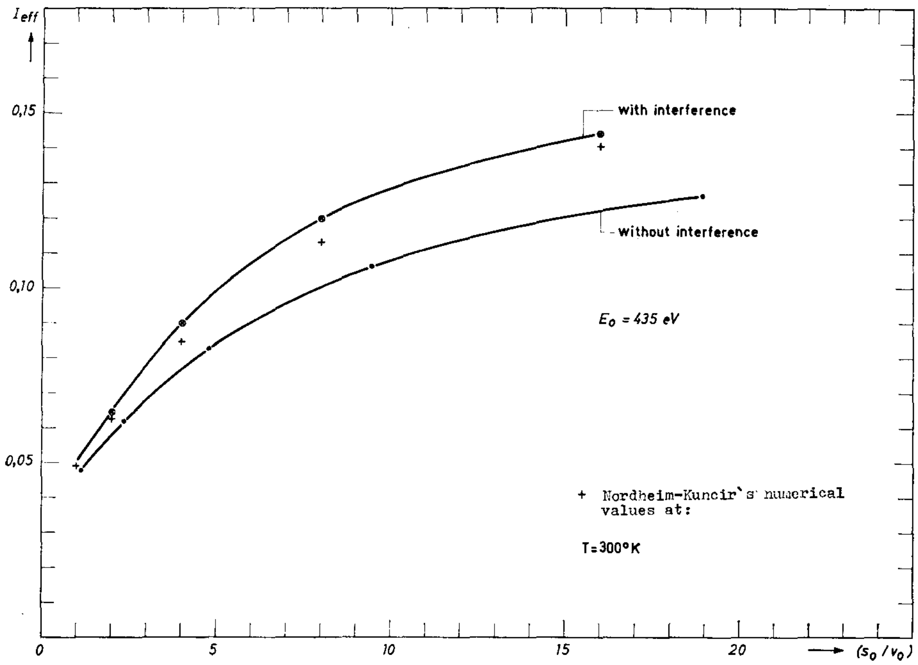
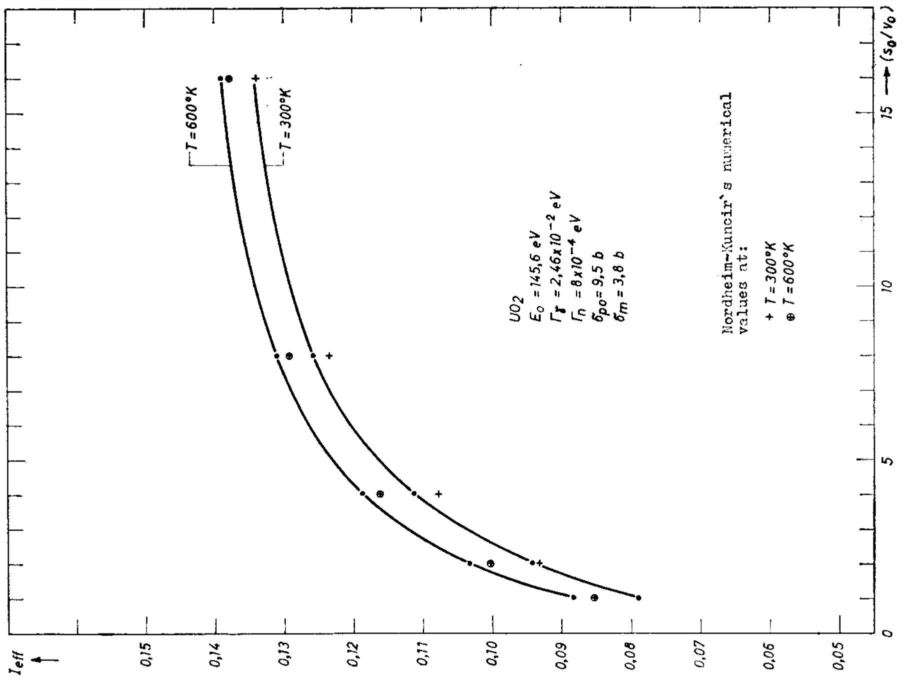
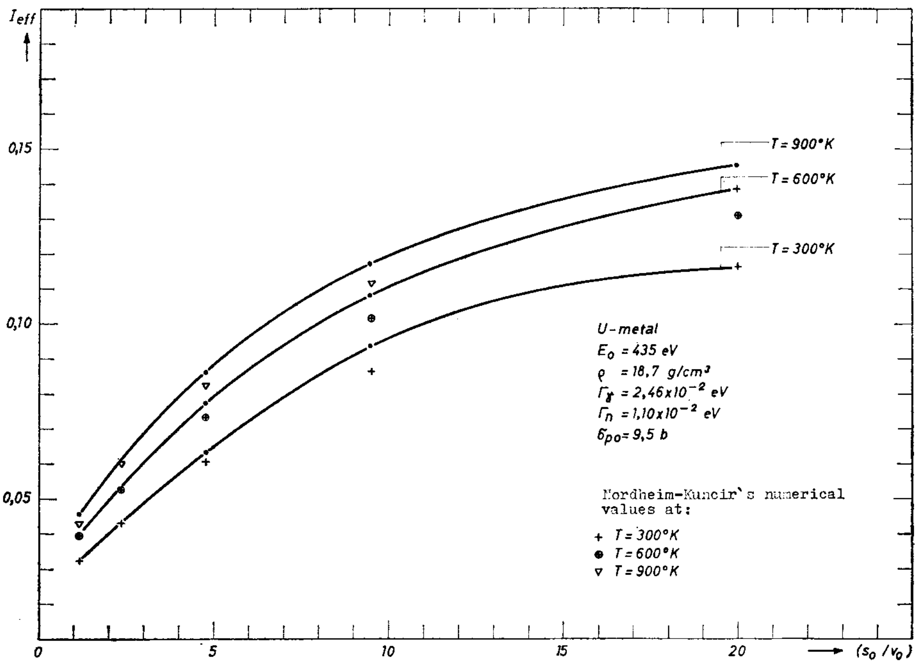

I_{eff}
0,15
0,10
0,05
with interference
without interference
E_o = 435 eV
+ Nordheim-Kuncir's numerical values at:
T=300°K
0
5
10
15
20
(s_o/v_o)

FIG.7



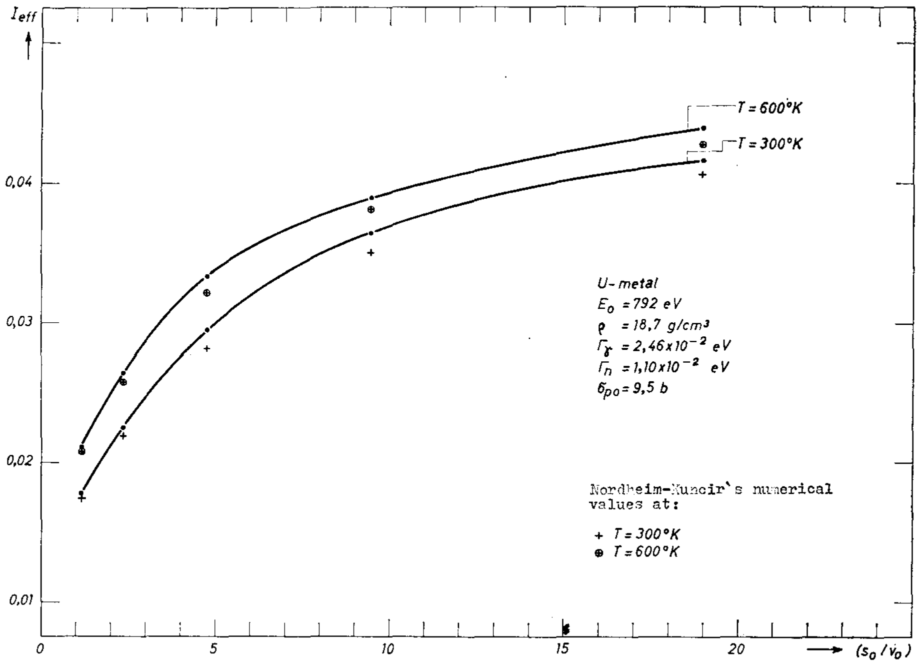
T = 600°K
T = 300°K
UO2
E0 = 145,6 eV
Γγ = 2,46x10^-2 eV
Γn = 8x10^-4 eV
σpo = 9,5 b
σm = 3,8 b
Nordheim-Kuncir's numerical values at:
+ T = 300°K
⊕ T = 600°K
Ieff
0,15
0,14
0,13
0,12
0,11
0,10
0,09
0,08
0,07
0,06
0,05
0
5
10
15
(s0/v0)

FIG. 8



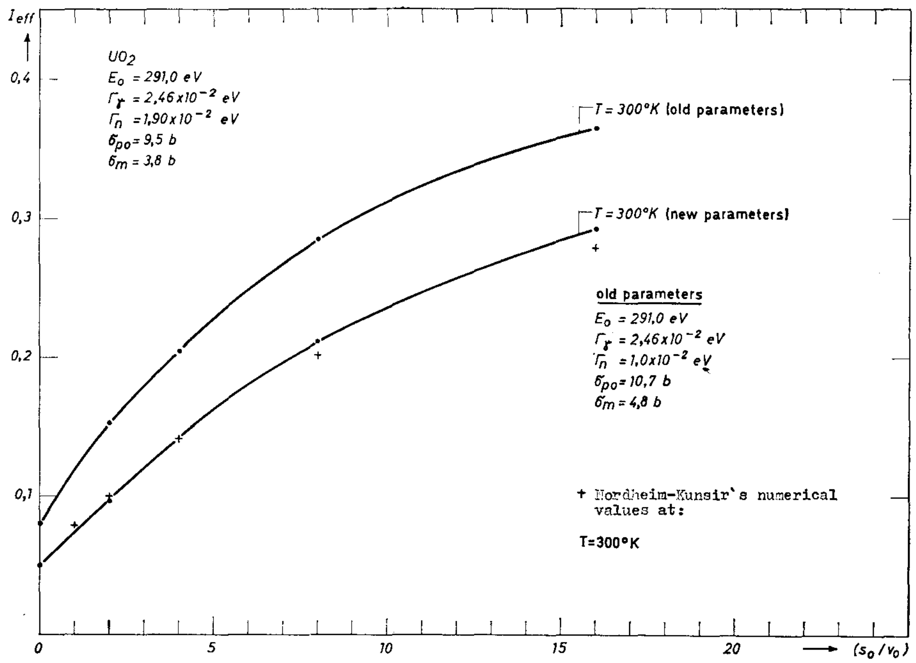
Ieff
0,15
0,10
0,05
T = 900°K
T = 600°K
T = 300°K
U-metal
E0 = 435 eV
ρ = 18,7 g/cm³
Γγ = 2,46x10^-2 eV
Γn = 1,10x10^-2 eV
σpo = 9,5 b
Nordheim-Kuncir's numerical values at:
+ T = 300°K
● T = 600°K
▽ T = 900°K
0
5
10
15
20
(s0/v0)

FIG. 9



I_{eff}
0,04
0,03
0,02
0,01
0
5
10
15
20
(s_0/v_0)
T = 600°K
T = 300°K
U-metal
E_0 = 792 eV
ρ = 18,7 g/cm³
Γ_γ = 2,46x10⁻² eV
Γ_n = 1,10x10⁻² eV
σ_{po} = 9,5 b
Nordheim-Kuncir's numerical values at:
+ T = 300°K
⊕ T = 600°K

FIG. 10



I_{eff}
0,4
0,3
0,2
0,1
0
5
10
15
20
(s_0/v_0)
UO_2
E_0 = 291,0 eV
Γ_γ = 2,46x10⁻² eV
Γ_n = 1,90x10⁻² eV
σ_{po} = 9,5 b
σ_m = 3,8 b
T = 300°K (old parameters)
T = 300°K (new parameters)
old parameters
E_0 = 291,0 eV
Γ_γ = 2,46x10⁻² eV
Γ_n = 1,0x10⁻² eV
σ_{po} = 10,7 b
σ_m = 4,8 b
+ Nordheim-Kunsir's numerical values at:
T=300°K

FIG. 11

i) Numerical analysis of both (2.32) and (2.40) was made for more cases:

1° $E_0 = 792$ eV, UO_2, T = 300 °K; 600 °K; 900 °K

2° $E_0 = 490$ eV, UO_2, T = 300 °K; 600 °K

3° $E_0 = 90$ eV, UO_2, T = 300 °K; 600 °K

4° $E_0 = 435$ eV, UO_2, T = 300 °K. Calculation was made with and without interference correction.

5° $E_0 = 145{,}6$ eV, UO_2, T = 300 °K

6° $E_0 = 345$ eV, U-metal, T = 300 °K; 600 °K; 900 °K

7° $E_0 = 792$ eV; U-metal, T = 300 °K; 600 °K

8° $E_0 = 291{,}0$ eV, UO_2, T = 300 °K. Calculation was made with old [23] and new parameters [6].

In all cases the used resonance parameters were from [6], and the NORDHEIM—KUNCIR's [12] values are given in diagrams too, for the purpose of comparison.

2.5. Correction of the Effective Resonance Integral for Interference Between Potential and Resonance Scattering

The total cross section in the resonance is given by

$$\sigma_t = \frac{\Gamma_\gamma}{\Gamma}\sigma_0 \Psi + \frac{\Gamma_n}{\Gamma}\sigma_0 \Psi + \sigma_p + \sigma_{si} \tag{2.42}$$

where

$$\sigma_{si} = \left(\frac{\Gamma_n}{\Gamma}\sigma_0 \sigma_{po}\right)^{1/2}\chi \tag{2.43}$$

is the interference scattering cross section, and Ψ and χ are the cross section functions. In case this cross section is not negligible, correction to the effective resonance integral, i.e. to the functions $J(\xi, \beta)$ and $K(\xi, \beta)$ is made in the following way:

$$2\,J_1(\xi. \beta, \gamma) = \int_{-\infty}^{+\infty} \frac{\Psi}{\Psi + \beta + \gamma\chi}\,dx \tag{2.44}$$

$$2\,J(\xi, \beta) = \int_{-\infty}^{+\infty} \frac{\Psi}{\Psi + \beta}\,dx \tag{2.45}$$

where $\gamma = \left(\frac{\Gamma_n}{\Gamma}\frac{\sigma_{po}}{\sigma_0}\right)^{1/2}$.

If we form the difference

$$\Delta J_0 = \frac{\Psi}{\Psi + \beta + \Psi + \gamma\chi} - \frac{\Psi}{\Psi + \beta} = -\frac{\gamma\Psi\chi}{(\Psi + \beta)^2}\left(1 - \frac{\gamma\chi}{\Psi + \beta} + \frac{\gamma^2\chi^2}{(\Psi + \beta)^2} - \cdots\right) \tag{2.46}$$

then after integration of the first three terms we write

$$\Delta J = \int_{-\infty}^{+\infty} \Delta J_0\,dx = -\int_{-\infty}^{+\infty} \frac{\gamma\Psi\chi}{(\Psi + \beta)^2}\,dx + \gamma^2 \int_{-\infty}^{+\infty} \frac{\Psi\chi^2}{(\Psi + \beta)^2}\,dx. \tag{2.47}$$

Since the function Ψ is even in x and χ is odd, the first integral is zero, and we have

(2.48) $$J_1(\xi,\ \beta,\ \gamma) = J(\xi,\ \beta) + \gamma^2 \int_{-\infty}^{+\infty} \frac{\Psi\chi^2}{(\Psi+\beta)^2}\, dx.$$

In the similar manner we make the correction of $K(\xi,\ \beta)$ for the same effect. Considering the difference

(2.49) $$\Delta K_0 = \frac{\Psi(\Psi+\gamma\chi)}{(\Psi+\gamma\chi+\beta)^2} - \frac{\Psi^2}{(\Psi+\beta)^2}$$

and after rearrangement

(2.50) $$\Delta K_0 = \frac{-\gamma\Psi\chi(\Psi^2-\beta^2) - \gamma^2\chi^2\Psi^2}{(\Psi+\beta)^4}\left(1 - 2\frac{\gamma\chi}{\Psi+\beta} + \cdots\right).$$

Integrating and using the above mentioned properties of the Ψ and χ functions, ultimately we can write

(2.51) $$K(\xi,\beta,\gamma) \approx K(\xi,\beta) - \gamma^2 \Delta K_i$$

where

(2.52) $$\Delta K_i = \int_0^{\infty} \frac{\Psi^2\chi^2}{(\Psi+\beta)^4}\, dx.$$

It is interesting to note that intereference correction for the function $K(\xi,\ \beta)$ is negative.

2.6. Tabulation of the Necessary Functions for the Calculation of the Effective Resonance Integral

In order to use the expression (2.32) in practice it is necessary to have the value of the functions $J(\xi,\ \beta)$, $K(\xi,\ \beta)$, $q(\xi,\ \beta)$, $\Delta J_i(\xi,\ \beta)$, $\Delta K_i(\xi,\ \beta)$ and $Ki_3(x)$. The functions $J(\xi,\ \beta)$ and $K(\xi,\ \beta)$ are already tabulated in *AHN*. They are calculated here again because the *AHN* values for lower values of ξ are not accurate enough and the values of the functions for $\xi = 0.06$; 0.07; 0.08; 0.09 do not exist at all.

The calculation was performed in the following way: The usual expressions for the $\Psi(\xi,\ x)$ and $\chi(\xi,\ x)$ functions by substitution of

$$y = \operatorname{tg} u$$

were brought to a suitable form for the numerical calculation of

(2.53) $$\Psi(\xi,\ x) = \frac{\xi}{\sqrt{4\pi}} \int_{-\pi/2}^{+\pi/2} e^{-\frac{\xi^2}{4}(x-\operatorname{tg} u)^2}\, du$$

and

(2.54) $$\chi(\xi,\ x) = \frac{\xi}{\sqrt{4\pi}} \int_{-\pi/2}^{+\pi/2} 2\operatorname{tg} u\, e^{-\frac{\xi^2}{4}(x-\operatorname{tg} u)^2}\, du.$$

Inaccuracy of the values of the functions $J(\xi, \beta)$ and $K(\xi, \beta)$ for small values of ξ in *AHN*, emerge from the way of calculation of the function $\Psi(\xi, x)$ for small ξ. Namely, they calculate it using the known series for small ξ, and directly for larger ξ [**23**]. Here the calculation was performed directly by (2.53) and (2.54) for both functions.

Both functions were calculated for the same values of the arguments:

$$\text{For } \xi = 0.02\ (0.01)\ 0.09 \quad \text{and} \quad x = 0(3)300$$

and

$$\text{for } \xi = 0.15\ (0.05)\ 0.50 \quad \text{and} \quad x = 0\ (2)200.$$

On the basis of these values by a separate *IBM* program and using (2.31), (2.30a), (2.47) and (2.51) the functions $J(\xi, \beta)$, $K(\xi, \beta)$, $q(\xi, \beta)$, $\Delta J_i(\xi, \beta)$ and $\Delta Ki(\xi, \beta)$ were calculated. The functions $J(\xi, \beta)$ and $K(\xi, \beta)$ are corrected for $x > 300$ and $x > 200$ respectively, using the known fact that function $\Psi(\xi, x)$ for large x can be represented by natural Breit—Wigner shape. The Carlvik—Pershagen's rational polynomial was the basis for the calculation of the $Ki_3(x)$ function in the range $x = 0\ (0.01)\ 10$. All these functions are given in the Appendix.

3. CONCLUSION

This work presents the original derivation of the expression for I_{eff}, which, after introduction of the flat flux approximation, gives the known formula. Its advantage in comparison to the existing methods is in the fact that this assumption of the flat fluxes is not taken a priori, and it is shown where correction has to be introduced if it is to be abandoned. A new approximate expression for the penetrability factor G is given. It retains the nature of the functional dependence of the exact expression for G, while the vagueness of the ad hoc introduced Wigner's rational approximation is eliminated. Its numerical investigation shows that a good accuracy is achieved with small number of terms.

On the basis of this expression for G a new formula for I_{eff} is derived. Its analysis shows correct dependence on the S_0/V_0 ratio, as well as correct limiting values. Also, it inherently consists of both "standard" formulas. In comparison to Rothestein's method, which according to [**8**] is the best thing done in the analytical treatment of the problem of resonance absorption in the heterogeneous systems, our expression simultaneously treats Doppler's effect and better approximation for the penetrability factor G. In comparison to the *AHN* method, in which the surface term is given by the function $L(t, \xi, \beta)$, i.e. by the function of three independent variables, our surface term is given as a product of two functions, one with two and the other with one independent variable, which is more suitable for practical purposes. By redefinition of the parameters our formula can be brought, just as in *AHN*, to the form for wide resonances, and by Fortie's model of the rectangular resonance, to the intermediary type. Numerical analysis of the formula for narrow resonances shows a good agreement with the pure numerical results, while its application to wide resonances has the same disadvantage as *AHN* does.

An original method of the correction of I_{eff} for the effect of interference between potential and resonance scattering is given.

In the numerical appendices all necessary functions for the application of the formula are given. These functions are tabulated more extensively and accurately than in other literature sources.

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$J(\xi, \beta) \quad \beta = 2^k \times 10^{-5}$

k/ξ	0.02	0.03	0.04	0.05	0.06	0.07	0.08
4.0	0.240024D 03	0.187668D 03	0.163298D 03	0.150267D 03	0.142589D 03	0.137821D 03	0.134589D
4.5	0.224242D 03	0.172264D 03	0.147384D 03	0.133676D 03	0.125390D 03	0.120115D 03	0.116494D
5.0	0.209813D 03	0.159031D 03	0.134104D 03	0.119999D 03	0.111268D 03	0.105575D 03	0.101612D
5.5	0.196230D 03	0.147363D 03	0.122821D 03	0.108601D 03	0.996050D 02	0.936087D 02	0.893722D
6.0	0.183101D 03	0.136776D 03	0.113007D 03	0.989419D 02	0.898664D 02	0.836950D 02	0.792713D
6.5	0.170120D 03	0.126887D 03	0.104237D 03	0.905729D 02	0.815982D 02	0.753847D 02	0.708686D
7.0	0.157058D 03	0.117398D 03	0.961709D 02	0.831294D 02	0.744226D 02	0.682961D 02	0.637845D
7.5	0.143757D 03	0.108080D 03	0.885416D 02	0.763191D 02	0.680312D 02	0.621121D 02	0.576988D
8.0	0.130135D 03	0.987686D 02	0.811459D 02	0.699128D 02	0.621774D 02	0.565742D 02	0.523463D
8.5	0.116198D 03	0.893623D 02	0.738367D 02	0.637365D 02	0.566686D 02	0.514766D 02	0.475125D
9.0	0.102056D 03	0.798262D 02	0.665219D 02	0.576665D 02	0.513610D 02	0.466610D 02	0.430284D
9.5	0.879299D 02	0.702003D 02	0.591668D 02	0.516288D 02	0.461560D 02	0.420120D 02	0.387670D
10.0	0.741439D 02	0.606049D 02	0.517986D 02	0.456006D 02	0.410006D 02	0.374562D 02	0.346402D
10.5	0.610881D 02	0.512355D 02	0.445083D 02	0.396130D 02	0.358885D 02	0.329621D 02	0.305990D
11.0	0.491514D 02	0.423395D 02	0.374442D 02	0.337501D 02	0.308611D 02	0.285413D 02	0.266334D
11.5	0.386452D 02	0.341737D 02	0.307905D 02	0.281385D 02	0.260023D 02	0.242461D 02	0.227721D
12.0	0.297445D 02	0.269522D 02	0.247329D 02	0.229259D 02	0.214255D 02	0.201612D 02	0.190765D
12.5	0.224697D 02	0.208038D 02	0.194192D 02	0.182504D 02	0.172506D 02	0.163872D 02	0.156294D
13.0	0.167100D 02	0.157548D 02	0.149295D 02	0.142098D 02	0.135769D 02	0.130175D 02	0.125152D
13.5	0.122704D 02	0.117406D 02	0.112677D 02	0.108438D 02	0.104616D 02	0.101170D 02	0.980044D
14.0	0.892166D 01	0.863552D 01	0.837338D 01	0.813289D 01	0.791160D 01	0.770857D 01	0.751817D
14.5	0.643818D 01	0.628683D 01	0.614529D 01	0.601305D 01	0.588929D 01	0.577428D 01	0.566424D
15.0	0.461997D 01	0.454120D 01	0.446632D 01	0.439536D 01	0.432808D 01	0.426501D 01	0.420348D
15.5	0.330156D 01	0.326106D 01	0.322206D 01	0.318471D 01	0.314892D 01	0.311525D 01	0.308171D
16.0	0.235230D 01	0.233167D 01	0.231160D 01	0.229221D 01	0.227350D 01	0.225590D 01	0.223797D
16.5	0.167233D 01	0.166191D 01	0.165166D 01	0.164171D 01	0.163204D 01	0.162301D 01	0.161355D
17.0	0.118707D 01	0.118183D 01	0.117664D 01	0.117157D 01	0.116662D 01	0.116206D 01	0.115711D
17.5	0.841678D 00	0.839064D 00	0.836445D 00	0.833876D 00	0.831364D 00	0.829094D 00	0.826508D
18.0	0.596309D 00	0.595013D 00	0.593696D 00	0.592401D 00	0.591132D 00	0.590022D 00	0.588670D
18.5	0.422232D 00	0.421595D 00	0.420936D 00	0.420285D 00	0.419646D 00	0.419114D 00	0.418404D
19.0	0.298854D 00	0.298543D 00	0.298214D 00	0.297888D 00	0.297567D 00	0.297320D 00	0.296943D
19.5	0.211467D 00	0.211317D 00	0.211154D 00	0.210991D 00	0.210830D 00	0.210721D 00	0.210518D
20.0	0.149602D 00	0.149532D 00	0.149451D 00	0.149370D 00	0.149290D 00	0.149245D 00	0.149134D

k/ξ	0.15	0.20	0.25	0.30	0.35	0.40	0.45
4.0	0.126932D 03	0.125638D 03	0.125114D 03	0.124812D 03	0.124628D 03	0.124561D 03	0.124468D
4.5	0.107698D 03	0.106161D 03	0.105538D 03	0.105177D 03	0.104964D 03	0.104866D 03	0.104760D
5.0	0.916766D 02	0.898702D 02	0.891338D 02	0.887069D 02	0.884573D 02	0.883252D 02	0.882046D
5.5	0.783629D 02	0.762710D 02	0.754043D 02	0.749023D 02	0.746088D 02	0.744409D 02	0.743026D
6.0	0.673257D 02	0.649430D 02	0.639302D 02	0.633444D 02	0.630002D 02	0.627936D 02	0.626345D
6.5	0.581904D 02	0.555272D 02	0.543568D 02	0.536802D 02	0.532790D 02	0.530309D 02	0.528476D
7.0	0.506291D 02	0.477139D 02	0.463826D 02	0.456108D 02	0.451479D 02	0.448558D 02	0.446451D
7.5	0.443536D 02	0.412332D 02	0.397489D 02	0.388822D 02	0.383553D 02	0.380173D 02	0.377766D
8.0	0.391109D 02	0.358486D 02	0.342317D 02	0.332765D 02	0.326868D 02	0.323027D 02	0.320305D
8.5	0.346814D 02	0.313526D 02	0.296357D 02	0.286054D 02	0.279585D 02	0.275306D 02	0.272268D
9.0	0.308777D 02	0.275648D 02	0.257900D 02	0.247052D 02	0.240115D 02	0.235451D 02	0.232116D
9.5	0.275429D 02	0.243296D 02	0.225454D 02	0.214328D 02	0.207075D 02	0.202109D 02	0.198523D
10.0	0.245501D 02	0.215162D 02	0.197735D 02	0.186638D 02	0.179257D 02	0.174106D 02	0.170343D
10.5	0.218003D 02	0.190173D 02	0.173658D 02	0.162914D 02	0.155615D 02	0.150421D 02	0.146578D
11.0	0.192221D 02	0.167489D 02	0.152335D 02	0.142253D 02	0.135253D 02	0.130174D 02	0.126370D
11.5	0.167707D 02	0.146493D 02	0.133068D 02	0.123924D 02	0.117430D 02	0.112629D 02	0.108985D
12.0	0.144278D 02	0.126796D 02	0.115354D 02	0.107363D 02	0.101554D 02	0.971812D 01	0.938139D
12.5	0.121996D 02	0.108221D 02	0.988828D 01	0.921801D 01	0.871921D 01	0.833672D 01	0.803743D
13.0	0.101121D 02	0.907909D 01	0.835237D 01	0.781540D 01	0.740605D 01	0.708603D 01	0.683119D
13.5	0.820260D 01	0.746769D 01	0.693063D 01	0.652159D 01	0.620193D 01	0.594690D 01	0.574004D
14.0	0.650831D 01	0.601289D 01	0.563694D 01	0.534165D 01	0.510503D 01	0.491223D 01	0.475296D
14.5	0.505513D 01	0.473825D 01	0.448903D 01	0.428731D 01	0.412163D 01	0.398380D 01	0.386794D
15.0	0.385087D 01	0.365781D 01	0.350100D 01	0.337046D 01	0.326072D 01	0.316764D 01	0.308814D
15.5	0.288451D 01	0.277183D 01	0.267775D 01	0.259743D 01	0.252850D 01	0.246902D 01	0.241752D
16.0	0.213073D 01	0.206728D 01	0.201313D 01	0.196588D 01	0.192461D 01	0.188849D 01	0.185689D
16.5	0.155649D 01	0.152179D 01	0.149168D 01	0.146492D 01	0.144122D 01	0.142026D 01	0.140179D
17.0	0.112725D 01	0.110870D 01	0.109241D 01	0.107772D 01	0.106458D 01	0.105287D 01	0.104253D
17.5	0.811081D 00	0.801326D 00	0.792710D 00	0.784839D 00	0.777750D 00	0.771423D 01	0.765838D
18.0	0.580771D 00	0.575705D 00	0.571224D 00	0.567086D 00	0.563347D 00	0.560018D 00	0.557101D
18.5	0.414387D 00	0.411777D 00	0.409478D 00	0.407333D 00	0.405396D 00	0.403685D 00	0.402207D
19.0	0.294910D 00	0.293573D 00	0.292406D 00	0.291306D 00	0.290317D 00	0.289456D 00	0.288732D
19.5	0.209492D 00	0.208809D 00	0.208222D 00	0.207663D 00	0.207164D 00	0.206741D 00	0.206400D
20.0	0.148618D 00	0.148268D 00	0.147976D 00	0.147693D 00	0.147445D 00	0.147242D 00	0.147091D

$K(\xi, \beta) \quad \beta = 2^k \times 10^{-5}$

k/ξ	0.02	0.03	0.04	0.05	0.06	0.07	0.08
4.0	0.191511D 03	0.138964D 03	0.112431D 03	0.970572D 02	0.874231D 02	0.810947D 02	0.766928D
4.5	0.180703D 03	0.131045D 03	0.105374D 03	0.901111D 02	0.802935D 02	0.736704D 02	0.689812D
5.0	0.169498D 03	0.123250D 03	0.988313D 02	0.839917D 02	0.742296D 02	0.674865D 02	0.626260D
5.5	0.157801D 03	0.115404D 03	0.925539D 02	0.784000D 02	0.689080D 02	0.622166D 02	0.573103D
6.0	0.145529D 03	0.107369D 03	0.863502D 02	0.730973D 02	0.640580D 02	0.575734D 02	0.527437D
6.5	0.132626D 03	0.990416D 02	0.800771D 02	0.679025D 02	0.594669D 02	0.533215D 02	0.486789D
7.0	0.119075D 03	0.903510D 02	0.736315D 02	0.626826D 02	0.549748D 02	0.492782D 02	0.449176D
7.5	0.104931D 03	0.812638D 02	0.669452D 02	0.573443D 02	0.504664D 02	0.453081D 02	0.413081D
8.0	0.903485D 02	0.717966D 02	0.599869D 02	0.518294D 02	0.458632D 02	0.413157D 02	0.377404D
8.5	0.756093D 02	0.620328D 02	0.527688D 02	0.461155D 02	0.411210D 02	0.372396D 02	0.341395D
9.0	0.611396D 02	0.521425D 02	0.453596D 02	0.402220D 02	0.362305D 02	0.330513D 02	0.304625D
9.5	0.474836D 02	0.423924D 02	0.378968D 02	0.342193D 02	0.312233D 02	0.287574D 02	0.266981D
10.0	0.352245D 02	0.331346D 02	0.305919D 02	0.282373D 02	0.261792D 02	0.244046D 02	0.228704D
10.5	0.248560D 02	0.247578D 02	0.237164D 02	0.224641D 02	0.212294D 02	0.200848D 02	0.190429D
11.0	0.166524D 02	0.176063D 02	0.175607D 02	0.171271D 02	0.165489D 02	0.159325D 02	0.153191D
11.5	0.106004D 02	0.118890D 02	0.123682D 02	0.124506D 02	0.123309D 02	0.121095D 02	0.118335D
12.0	0.643375D 01	0.762612D 01	0.827078D 01	0.860119D 01	0.874421D 01	0.877424D 01	0.872953D
12.5	0.374346D 01	0.466090D 01	0.525579D 01	0.564170D 01	0.588827D 01	0.604128D 01	0.612522D
13.0	0.210182D 01	0.272836D 01	0.318496D 01	0.352020D 01	0.376707D 01	0.394978D 01	0.408098D
13.5	0.114646D 01	0.153958D 01	0.185070D 01	0.209853D 01	0.229690D 01	0.245712D 01	0.258443D
14.0	0.611322D 00	0.843145D 00	0.103797D 01	0.120244D 01	0.134181D 01	0.146092D 01	0.156135D
14.5	0.320373D 00	0.450983D 00	0.565682D 00	0.666698D 00	0.755892D 00	0.835231D 00	0.904893D
15.0	0.165734D 00	0.236900D 00	0.301435D 00	0.360063D 00	0.413409D 00	0.462265D 00	0.506428D
15.5	0.849219D-01	0.122766D 00	0.157893D 00	0.190535D 00	0.220896D 00	0.249302D 00	0.275531D
16.0	0.432121D-01	0.629852D-01	0.816503D-01	0.992814D-01	0.115946D 00	0.131782D 00	0.146632D
16.5	0.218781D-01	0.320794D-01	0.418261D-01	0.511424D-01	0.600503D-01	0.686118D-01	0.767312D-
17.0	0.110369D-01	0.162525D-01	0.212788D-01	0.261242D-01	0.307959D-01	0.353227D-01	0.396508D-
17.5	0.555344D-02	0.820281D-02	0.107718D-01	0.132634D-01	0.156800D-01	0.180355D-01	0.203008D-
18.0	0.278921D-02	0.412884D-02	0.543354D-02	0.670437D-02	0.794226D-02	0.915387D-02	0.103240D-
18.5	0.139905D-02	0.207421D-02	0.273381D-02	0.337827D-02	0.400792D-02	0.462606D-02	0.522480D-
19.0	0.701108D-03	0.104059D-02	0.137299D-02	0.169846D-02	0.201714D-02	0.233066D-02	0.263498D-
19.5	0.351120D-03	0.521539D-03	0.688661D-03	0.852554D-03	0.101327D-02	0.117163D-02	0.132557D-
20.0	0.175764D-03	0.261214D-03	0.345104D-03	0.427463D-03	0.508315D-03	0.588062D-03	0.665671D-

k/ξ	0.15	0.20	0.25	0.30	0.35	0.40	0.45
4.0	0.648908D 02	0.629312D 02	0.621434D 02	0.616836D 02	0.614244D 02	0.612693D 02	0.611433D
4.5	0.563800D 02	0.540999D 02	0.531728D 02	0.526325D 02	0.523241D 02	0.521318D 02	0.519872D
5.0	0.492060D 02	0.465956D 02	0.455078D 02	0.448766D 02	0.445110D 02	0.442788D 02	0.441117D
5.5	0.432341D 02	0.403039D 02	0.390372D 02	0.383057D 02	0.378756D 02	0.376002D 02	0.374066D
6.0	0.382938D 02	0.350785D 02	0.336245D 02	0.327860D 02	0.322859D 02	0.319637D 02	0.317396D
6.5	0.342007D 02	0.307603D 02	0.291256D 02	0.281788D 02	0.276055D 02	0.272338D 02	0.269760D
7.0	0.307718D 02	0.271885D 02	0.253984D 02	0.243497D 02	0.237041D 02	0.232815D 02	0.229878D
7.5	0.278372D 02	0.242091D 02	0.223067D 02	0.211720D 02	0.204603D 02	0.199887D 02	0.196587D
8.0	0.252484D 02	0.216808D 02	0.197230D 02	0.185282D 02	0.177630D 02	0.172481D 02	0.168837D
8.5	0.228829D 02	0.194792D 02	0.175305D 02	0.163101D 02	0.155107D 02	0.149625D 02	0.145690D
9.0	0.206455D 02	0.174998D 02	0.156258D 02	0.144199D 02	0.136109D 02	0.130439D 02	0.126301D
9.5	0.184676D 02	0.156598D 02	0.139216D 02	0.127717D 02	0.119808D 02	0.114127D 02	0.109910D
10.0	0.163057D 02	0.138978D 02	0.123481D 02	0.112933D 02	0.105479D 02	0.999874D 01	0.958434D
10.5	0.141413D 02	0.121743D 02	0.108538D 02	0.992701D 01	0.925232D 01	0.874270D 01	0.835191D
11.0	0.119809D 02	0.104712D 02	0.940618D 01	0.863130D 01	0.804832D 01	0.759737D 01	0.724551D
11.5	0.985664D 01	0.879205D 01	0.799119D 01	0.738160D 01	0.690576D 01	0.652856D 01	0.622786D
12.0	0.782471D 01	0.716151D 01	0.661406D 01	0.617126D 01	0.581027D 01	0.551559D 01	0.527392D
12.5	0.595707D 01	0.562249D 01	0.529811D 01	0.501084D 01	0.476274D 01	0.455179D 01	0.437216D
13.0	0.432655D 01	0.422865D 01	0.408075D 01	0.392530D 01	0.377789D 01	0.364433D 01	0.352466D
13.5	0.298733D 01	0.303153D 01	0.300474D 01	0.294807D 01	0.288071D 01	0.281167D 01	0.274461D
14.0	0.195899D 01	0.206547D 01	0.210611D 01	0.211195D 01	0.209954D 01	0.207771D 01	0.205139D
14.5	0.122226D 01	0.133697D 01	0.140258D 01	0.143864D 01	0.145673D 01	0.146371D 01	0.146397D
15.0	0.728674D 00	0.824185D 00	0.888197D 00	0.931390D 00	0.960690D 00	0.980579D 00	0.994177D
15.5	0.417564D 00	0.486170D 00	0.536700D 00	0.574389D 00	0.602908D 00	0.624810D 00	0.641969D
16.0	0.231546D 00	0.276125D 00	0.311146D 00	0.338985D 00	0.361422D 00	0.379764D 00	0.394998D
16.5	0.125063D 00	0.152016D 00	0.174194D 00	0.192623D 00	0.208120D 00	0.221302D 00	0.232642D
17.0	0.661856D-01	0.816461D-01	0.948028D-01	0.106092D 00	0.115874D 00	0.124428D 00	0.131963D
17.5	0.344896D-01	0.430233D-01	0.504654D-01	0.570011D-01	0.627887D-01	0.679504D-01	0.725744D
18.0	0.177674D-01	0.223478D-01	0.264137D-01	0.300452D-01	0.333119D-01	0.362670D-01	0.389465D
18.5	0.907622D-02	0.114854D-01	0.136516D-01	0.156099D-01	0.173917D-01	0.190200D-01	0.205095D
19.0	0.460820D-02	0.585707D-02	0.699032D-02	0.802385D-02	0.897190D-02	0.984470D-02	0.106481D
19.5	0.232942D-02	0.297009D-02	0.355528D-02	0.409232D-02	0.458784D-02	0.504643D-02	0.547047D
20.0	0.117381D-02	0.150003D-02	0.179941D-02	0.207538D-02	0.233108D-02	0.256862D-02	0.278897D

$q(\xi, \beta)$ $\beta = 2^k \times 10^{-5}$

k/ξ	0.02	0.03	0.04	0.05	0.06	0.07	0.08
4.0	0.628444D-02	0.870814D-02	0.107905D-01	0.125165D-01	0.139069D-01	0.149995D-01	0.158657D-
4.5	0.660670D-02	0.918072D-02	0.114593D-01	0.134273D-01	0.150874D-01	0.164565D-01	0.175845D-
5.0	0.696746D-02	0.968543D-02	0.121419D-01	0.143293D-01	0.162433D-01	0.178875D-01	0.192917D-
5.5	0.737609D-02	0.102364D-01	0.128579D-01	0.152436D-01	0.173898D-01	0.192944D-01	0.209724D-
6.0	0.784490D-02	0.108500D-01	0.136293D-01	0.161971D-01	0.185539D-01	0.206976D-01	0.226350D-
6.5	0.839030D-02	0.115457D-01	0.144811D-01	0.172205D-01	0.197706D-01	0.221320D-01	0.243087D-
7.0	0.903473D-02	0.123483D-01	0.154420D-01	0.183485D-01	0.210802D-01	0.236421D-01	0.260383D-
7.5	0.980944D-02	0.132905D-01	0.165481D-01	0.196218D-01	0.225294D-01	0.252801D-01	0.278800D-
8.0	0.107589D-01	0.144173D-01	0.178464D-01	0.210910D-01	0.241737D-01	0.271072D-01	0.299004D-
8.5	0.119469D-01	0.157917D-01	0.194007D-01	0.228225D-01	0.260832D-01	0.291982D-01	0.321798D-
9.0	0.134660D-01	0.175040D-01	0.213009D-01	0.249070D-01	0.283509D-01	0.316497D-01	0.348190D-
9.5	0.154507D-01	0.196846D-01	0.236755D-01	0.274726D-01	0.311055D-01	0.345923D-01	0.379517D-
10.0	0.180954D-01	0.225229D-01	0.267101D-01	0.307026D-01	0.345295D-01	0.382083D-01	0.417618D-
10.5	0.216808D-01	0.262931D-01	0.306738D-01	0.348622D-01	0.388853D-01	0.427584D-01	0.465094D-
11.0	0.266090D-01	0.313906D-01	0.359560D-01	0.403356D-01	0.445528D-01	0.486186D-01	0.525687D-
11.5	0.334530D-01	0.383828D-01	0.431179D-01	0.476779D-01	0.520815D-01	0.563328D-01	0.604800D-
12.0	0.430266D-01	0.480782D-01	0.529630D-01	0.576868D-01	0.622634D-01	0.666858D-01	0.710245D-
12.5	0.564821D-01	0.616250D-01	0.666358D-01	0.715026D-01	0.762334D-01	0.808047D-01	0.853261D-
13.0	0.754506D-01	0.806486D-01	0.857600D-01	0.907459D-01	0.956088D-01	0.100298D 00	0.104993D
13.5	0.102242D 00	0.107448D 00	0.112633D 00	0.117713D 00	0.122684D 00	0.127449D 00	0.132314D
14.0	0.140131D 00	0.145272D 00	0.150503D 00	0.155653D 00	0.160708D 00	0.165488D 00	0.170535D
14.5	0.193764D 00	0.198720D 00	0.203961D 00	0.209156D 00	0.214270D 00	0.218974D 00	0.224248D
15.0	0.269755D 00	0.274299D 00	0.279501D 00	0.284709D 00	0.289858D 00	0.294333D 00	0.299938D
15.5	0.377535D 00	0.381230D 00	0.386308D 00	0.391489D 00	0.396642D 00	0.400608D 00	0.406772D
16.0	0.530602D 00	0.532591D 00	0.537390D 00	0.542475D 00	0.547587D 00	0.550514D 00	0.557702D
16.5	0.748376D 00	0.746953D 00	0.751174D 00	0.756040D 00	0.761035D 00	0.761890D 00	0.771034D
17.0	0.105897D 01	0.105074D 01	0.105380D 01	0.105820D 01	0.106294D 01	0.105969D 01	0.107265D
17.5	0.150347D 01	0.148166D 01	0.148237D 01	0.148585D 01	0.149006D 01	0.147868D 01	0.149914D
18.0	0.214258D 01	0.209365D 01	0.208970D 01	0.209131D 01	0.209445D 01	0.206694D 01	0.210225D
18.5	0.306740D 01	0.296434D 01	0.295105D 01	0.294894D 01	0.294994D 01	0.289034D 01	0.295517D
19.0	0.441726D 01	0.420605D 01	0.417412D 01	0.416457D 01	0.416130D 01	0.403780D 01	0.416139D
19.5	0.641018D 01	0.598286D 01	0.591368D 01	0.588927D 01	0.587746D 01	0.562657D 01	0.586729D
20.0	0.939637D 01	0.853712D 01	0.839351D 01	0.833942D 01	0.831056D 01	0.780546D 01	0.827991D

k/ξ	0.15	0.20	0.25	0.30	0.35	0.40	0.45
4.0	0.187661D-01	0.193531D-01	0.195995D-01	0.197462D-01	0.198299D-01	0.198802D-01	0.199214D
4.5	0.215424D-01	0.224554D-01	0.228490D-01	0.230848D-01	0.232215D-01	0.233076D-01	0.233727D
5.0	0.246039D-01	0.259920D-01	0.266174D-01	0.269941D-01	0.272173D-01	0.273608D-01	0.274651D
5.5	0.278912D-01	0.299373D-01	0.309166D-01	0.315117D-01	0.318723D-01	0.321073D-01	0.322749D
6.0	0.313330D-01	0.342391D-01	0.357348D-01	0.366577D-01	0.372309D-01	0.376094D-01	0.378775D
6.5	0.348628D-01	0.388237D-01	0.410313D-01	0.424270D-01	0.433184D-01	0.439162D-01	0.443407D
7.0	0.384379D-01	0.436116D-01	0.467381D-01	0.487829D-01	0.501312D-01	0.510537D-01	0.517151D
7.5	0.420532D-01	0.485387D-01	0.527728D-01	0.556593D-01	0.576321D-01	0.590159D-01	0.600240D
8.0	0.457480D-01	0.535785D-01	0.590616D-01	0.629738D-01	0.657534D-01	0.677613D-01	0.692560D
8.5	0.496041D-01	0.587577D-01	0.655677D-01	0.706533D-01	0.744132D-01	0.772210D-01	0.793657D
9.0	0.537421D-01	0.641638D-01	0.723158D-01	0.786661D-01	0.835464D-01	0.873224D-01	0.902891D
9.5	0.583221D-01	0.699460D-01	0.794081D-01	0.870530D-01	0.931442D-01	0.980283D-01	0.101974D
10.0	0.635517D-01	0.763184D-01	0.870316D-01	0.959507D-01	0.103294D 00	0.109381D 00	0.114426D
10.5	0.697070D-01	0.835699D-01	0.954671D-01	0.105611D 00	0.114210D 00	0.121543D 00	0.127754D
11.0	0.771659D-01	0.920905D-01	0.105109D 00	0.116424D 00	0.126256D 00	0.134825D 00	0.142217D
11.5	0.864603D-01	0.102418D 00	0.116504D 00	0.128943D 00	0.139967D 00	0.149726D 00	0.158287D
12.0	0.983498D-01	0.115311D 00	0.130420D 00	0.143944D 00	0.156103D 00	0.166994D 00	0.176700D
12.5	0.113928D 00	0.131854D 00	0.147945D 00	0.162513D 00	0.175743D 00	0.187711D 00	0.198537D
13.0	0.134771D 00	0.153608D 00	0.170633D 00	0.186192D 00	0.200429D 00	0.213424D 00	0.225322D
13.5	0.163137D 00	0.182816D 00	0.200709D 00	0.217193D 00	0.232368D 00	0.246327D 00	0.259204D
14.0	0.202254D 00	0.222693D 00	0.241365D 00	0.258690D 00	0.274722D 00	0.289547D 00	0.303254D
14.5	0.256709D 00	0.277816D 00	0.297155D 00	0.315224D 00	0.332007D 00	0.347559D 00	0.361901D
15.0	0.332994D 00	0.354690D 00	0.374562D 00	0.393263D 00	0.410667D 00	0.426766D 00	0.441510D
15.5	0.440292D 00	0.462521D 00	0.482771D 00	0.501988D 00	0.519857D 00	0.536283D 00	0.551145D
16.0	0.591575D 00	0.614339D 00	0.634773D 00	0.654387D 00	0.672538D 00	0.689008D 00	0.703615D
16.5	0.805172D 00	0.828572D 00	0.848922D 00	0.868818D 00	0.887017D 00	0.903126D 00	0.916899D
17.0	0.110698D 01	0.113131D 01	0.115115D 01	0.117120D 01	0.118911D 01	0.120420D 01	0.121614D
17.5	0.153362D 01	0.155951D 01	0.157810D 01	0.159816D 01	0.161522D 01	0.162809D 01	0.163638D
18.0	0.213684D 01	0.216562D 01	0.218155D 01	0.220141D 01	0.221661D 01	0.222503D 01	0.222615D
18.5	0.298984D 01	0.302419D 01	0.303474D 01	0.305403D 01	0.306542D 01	0.306504D 01	0.305217D
19.0	0.419613D 01	0.424139D 01	0.424119D 01	0.425925D 01	0.426303D 01	0.424531D 01	0.420503D
19.5	0.590210D 01	0.596895D 01	0.594734D 01	0.596286D 01	0.595156D 01	0.589960D 01	0.580536D
20.0	0.831479D 01	0.842450D 01	0.836028D 01	0.837072D 01	0.832946D 01	0.820972D 01	0.800884D

$\Delta J_i(\xi, \beta) \quad \beta = 2^k \times 10^{-5}$

k/ξ	0.02	0.03	0.04	0.05	0.06	0.07	0.08
4.0	0.486944D 05	0.872519D 05	0.128216D 06	0.167778D 06	0.202968D 06	0.232709D 06	0.257116D
4.5	0.278782D 05	0.459697D 05	0.660667D 05	0.866618D 05	0.106167D 06	0.123707D 06	0.138812D
5.0	0.160488D 05	0.245556D 05	0.342181D 05	0.445504D 05	0.548496D 05	0.646260D 05	0.734569D
5.5	0.931920D 04	0.133722D 05	0.179792D 05	0.230310D 05	0.282616D 05	0.334499D 05	0.383456D
6 0	0.546275D 04	0.744015D 04	0.964462D 04	0.120809D 05	0.146650D 05	0.173128D 05	0.199047D
6.5	0.322790D 04	0.422587D 04	0.529733D 04	0.647094D 04	0.772810D 04	0.904284D 04	0.103661D
7.0	0.191609D 04	0.244201D 04	0.297741D 04	0.354952D 04	0.415982D 04	0.480319D 04	0.546246D
7.5	0.113664D 04	0.142810D 04	0.170639D 04	0.199245D 04	0.229224D 04	0.260685D 04	0.293166D
8.0	0.669273D 03	0.839408D 03	0.991312D 03	0.113999D 04	0.129119D 04	0.144725D 04	0.160778D
8.5	0.388038D 03	0.491920D 03	0.579345D 03	0.660531D 03	0.739928D 03	0.819688D 03	0.900535D
9.0	0.219597D 03	0.284863D 03	0.337631D 03	0.384392D 03	0.428251D 03	0.470847D 03	0.513001D
9.5	0.120229D 03	0.161469D 03	0.194343D 03	0.222562D 03	0.248095D 03	0.272060D 03	0.295112D
10.0	0.631731D 02	0.887487D 02	0.109402D 03	0.126928D 03	0.142433D 03	0.156599D 03	0.169873D
10.5	0.316588D 02	0.468954D 02	0.596527D 02	0.705816D 02	0.801977D 02	0.888629D 02	0.968379D
11.0	0.150796D 02	0.236621D 02	0.312385D 02	0.379072D 02	0.438410D 02	0.491925D 02	0.540889D
11.5	0.682524D 01	0.113544D 02	0.156097D 02	0.195061D 02	0.230589D 02	0.263080D 02	0.292995D
12.0	0 294382D 01	0.517725D 01	0.741635D 01	0.956348D 01	0.115873D 02	0.134821D 02	0.152551D
12.5	0.121627D 01	0.224845D 01	0.334932D 01	0.445657D 01	0.553984D 01	0.658398D 01	0.758345D
13.0	0.484508D 00	0.934602D 00	0.144188D 01	0.197567D 01	0.251777D 01	0.305675D 01	0.358612D
13.5	0.187362D 00	0.374189D 00	0.594794D 00	0.836393D 00	0.109037D 01	0.135055D 01	0.161280D
14.0	0.707833D-01	0.145288D 00	0.236640D 00	0.340096D 00	0.452158D 00	0.570077D 00	0.691813D
14.5	0.262685D-01	0.550616D-01	0.914196D-01	0.133719D 00	0.180684D 00	0.231239D 00	0.284525D
15.0	0.961918D-02	0.204828D-01	0.345113D-01	0.511779D-01	0.700493D-01	0.907399D-01	0.112925D
15.5	0.348791D-02	0.751386D-02	0.128003D-01	0.191814D-01	0.265166D-01	0.346752D-01	0.435426D
16.0	0.125567D-02	0.272807D-02	0.468536D-02	0.707601D-02	0.985552D-02	0.129810D-01	0.164137D
16.5	0.449715D-03	0.983054D-03	0.169839D-02	0.257972D-02	0.361305D-02	0.478452D-02	0.608125D
17.0	0.160465D-03	0.352317D-03	0.611300D-03	0.932391D-03	0.131118D-02	0.174319D-02	0.222417D
17.5	0.571045D-04	0.125774D-03	0.218900D-03	0.334885D-03	0.472320D-03	0.629748D-03	0.805765D
18.0	0.202831D-04	0.447743D-04	0.780983D-04	0.119737D-03	0.169233D-03	0.226109D-03	0.289895D
18.5	0.719473D-05	0.159075D-04	0.277904D-04	0.426726D-04	0.604040D-04	0.808246D-04	0.103777D
19.0	0.254964D-05	0.564362D-05	0.987042D-05	0.151728D-04	0.215006D-04	0.287999D-04	0.370171D
19.5	0.902917D-06	0.200021D-05	0.350105D-05	0.538600D-05	0.763810D-05	0.102389D-04	0.131701D
20.0	0.319601D-06	0.708409D-06	0.124065D-05	0.190967D-05	0.270965D-05	0.363426D-05	0.467715D

k/ξ	0.15	0.20	0.25	0.30	0.35	0.40	0.45
4.0	0.315992D 06	0.334871D 06	0.341942D 06	0.346551D 06	0.349145D 06	0.350963D 06	0.352170D
4.5	0.186261D 06	0.200847D 06	0.206680D 06	0.210408D 06	0.212600D 06	0.214079D 06	0.215069D
5.0	0.107197D 06	0.118076D 06	0.122844D 06	0.125831D 06	0.127642D 06	0.128841D 06	0.129653D
5.5	0.603627D 05	0.681514D 05	0.719487D 05	0.743030D 05	0.757652D 05	0.767307D 05	0.773920D
6.0	0.333285D 05	0.386570D 05	0.415611D 05	0.433715D 05	0.445245D 05	0.452914D 05	0.458233D
6.5	0.180983D 05	0.215723D 05	0.236825D 05	0.250307D 05	0.259148D 05	0.265121D 05	0.269325D
7.0	0.970968D 04	0.118654D 05	0.133143D 05	0.142796D 05	0.149350D 05	0.153880D 05	0.157133D
7.5	0.517818D 04	0.645243D 04	0.739140D 04	0.805185D 04	0.851878D 04	0.885120D 04	0.909613D
8.0	0.276491D 04	0.348473D 04	0.406048D 04	0.449058D 04	0.480838D 04	0.504300D 04	0.522149D
8.5	0.148860D 04	0.187970D 04	0.221543D 04	0.248163D 04	0.268747D 04	0.284617D 04	0.297139D
9.0	0.812428D 03	0.101887D 04	0.120641D 04	0.136324D 04	0.149009D 04	0.159290D 04	0.167702D
9.5	0.450440D 03	0.557781D 03	0.659081D 03	0.747502D 03	0.822268D 03	0.886130D 03	0.939980D
10.0	0.253269D 03	0.309174D 03	0.362745D 03	0.410918D 03	0.453526D 03	0.491619D 03	0.524363D
10.5	0.143632D 03	0.173302D 03	0.201481D 03	0.227262D 03	0.251045D 03	0.272876D 03	0.291840D
11.0	0.814716D 02	0.977409D 02	0.112763D 03	0.126611D 03	0.139736D 03	0.151811D 03	0.162425D
11.5	0.457508D 02	0.550182D 02	0.632767D 02	0.708818D 02	0.780887D 02	0.846384D 02	0.905250D
12.0	0.251581D 02	0.306031D 02	0.353200D 02	0.396167D 02	0.435933D 02	0.471726D 02	0.504782D
12.5	0.134059D 02	0.166428D 02	0.194141D 02	0.218965D 02	0.241307D 02	0.261417D 02	0.280245D
13.0	0.686125D 01	0.875955D 01	0.103948D 02	0.118419D 02	0.131240D 02	0.142810D 02	0.153534D
13.5	0.335182D 01	0.442422D 01	0.536775D 01	0.620326D 01	0.694306D 01	0.760960D 01	0.821291D
14.0	0.155826D 01	0.213199D 01	0.265291D 01	0.312081D 01	0.353917D 01	0.391525D 01	0.424933D
14.5	0.689763D 00	0.977939D 00	0.124943D 01	0.149908D 01	0.172595D 01	0.193069D 01	0.211236D
15.0	0.291732D 00	0.427488D 00	0.560308D 00	0.685873D 00	0.802339D 00	0.908850D 00	0.100496D
15.5	0.118554D 00	0.178804D 00	0.239865D 00	0.299231D 00	0.355559D 00	0.408142D 00	0.456911D
16.0	0.465960D-01	0.719826D-01	0.985126D-01	0.124980D 00	0.150672D 00	0.175228D 00	0.198705D
16.5	0.178294D-01	0.280768D-01	0.390605D-01	0.502648D-01	0.613706D-01	0.722338D-01	0.829139D-
17.0	0.668143D-02	0.106792D-01	0.150512D-01	0.195930D-01	0.241766D-01	0.287520D-01	0.333575D-
17.5	0.246454D-02	0.398372D-02	0.567101D-02	0.744923D-02	0.927024D-02	0.111187D-01	0.130143D-
18.0	0.898447D-03	0.146444D-02	0.210042D-02	0.277810D-02	0.348009D-02	0.420207D-02	0.495312D-
18.5	0.324713D-03	0.532522D-03	0.768039D-03	0.102108D-02	0.128550D-02	0.156017D-02	0.184897D-
19.0	0.116623D-03	0.192110D-03	0.278200D-03	0.371260D-03	0.469139D-03	0.571575D-03	0.680125D-
19.5	0.416972D-04	0.689065D-04	0.100079D-03	0.133925D-03	0.169694D-03	0.207333D-03	0.247446D-
20.0	0.148602D-04	0.246134D-04	0.358236D-04	0.480345D-04	0.609838D-04	0.746644D-04	0.893041D-

$\Delta K_i(\xi, \beta) \quad \beta = 2^k \times 10^{-5}$

k/ξ	0.02	0.03	0.04	0.05	0.06	0.07	0.08
4.0	0.225227D 05	0.331206D 05	0.465774D 05	0.621314D 05	0.782598D 05	0.937532D 05	0.107574D
4.5	0.129783D 05	0.178885D 05	0.239878D 05	0.312509D 05	0.392040D 05	0.473430D 05	0.550585D
5.0	0.758686D 04	0.995904D 04	0.127585D 05	0.160887D 05	0.198518D 05	0.238875D 05	0.279258D
5.5	0.449001D 04	0.569244D 04	0.701660D 04	0.854839D 04	0.102910D 05	0.122111D 05	0.142139D
6.0	0.267962D 04	0.332116D 04	0.397512D 04	0.469627D 04	0.550339D 04	0.639798D 04	0.735471D
6.5	0.160388D 04	0.196485D 04	0.230503D 04	0.265839D 04	0.304033D 04	0.345776D 04	0.390718D
7.0	0.956187D 03	0.117038D 04	0.135772D 04	0.154021D 04	0.172837D 04	0.192780D 04	0.214027D
7.5	0.563053D 03	0.696431D 03	0.805787D 03	0.905893D 03	0.100381D 04	0.110333D 04	0.120671D
8.0	0.324280D 03	0.410335D 03	0.477658D 03	0.536086D 03	0.590363D 03	0.643046D 03	0.695805D
8.5	0.180642D 03	0.237002D 03	0.280125D 03	0.316159D 03	0.348213D 03	0.378013D 03	0.406670D
9.0	0.961653D 02	0.132693D 03	0.160815D 03	0.183908D 03	0.203866D 03	0.221806D 03	0.238457D
9.5	0.483343D 02	0.711508D 02	0.893298D 02	0.104333D 03	0.117166D 03	0.128479D 03	0.138729D
10.0	0.226852D 02	0.360926D 02	0.474255D 02	0.570312D 02	0.653167D 02	0.726011D 02	0.791331D
10.5	0.985853D 01	0.171252D 02	0.237707D 02	0.296639D 02	0.348797D 02	0.395257D 02	0.437081D
11.0	0.394922D 01	0.753248D 01	0.111245D 02	0.145056D 02	0.176200D 02	0.204692D 02	0.230781D
11.5	0.145864D 01	0.305537D 01	0.482027D 01	0.660029D 01	0.832451D 01	0.996176D 01	0.115031D
12.0	0.499140D 00	0.114233D 01	0.192535D 01	0.277481D 01	0.364543D 01	0.450967D 01	0.535253D
12.5	0.159579D 00	0.395302D 00	0.709177D 00	0.107494D 01	0.147224D 01	0.188595D 01	0.230574D
13.0	0.481619D-01	0.127621D 00	0.242080D 00	0.384538D 00	0.548152D 00	0.726830D 00	0.915679D
13.5	0.138702D-01	0.388307D-01	0.772312D-01	0.127847D 00	0.188976D 00	0.258759D 00	0.335452D
14.0	0.385011D-02	0.112558D-01	0.232682D-01	0.398746D-01	0.608120D-01	0.856629D-01	0.113951D
14.5	0.103905D-02	0.314019D-02	0.669175D-02	0.117922D-01	0.184533D-01	0.266215D-01	0.362029D
15.0	0.274567D-03	0.850703D-03	0.185552D-02	0.334181D-02	0.533758D-02	0.784985D-02	0.108699D
15.5	0.714347D-04	0.225434D-03	0.500360D-03	0.916200D-03	0.148663D-02	0.221948D-02	0.311763D
16.0	0.183760D-04	0.587737D-04	0.132140D-03	0.244966D-03	0.402235D-03	0.607433D-03	0.862668D
16.5	0.468858D-05	0.151419D-04	0.343641D-04	0.642866D-04	0.106492D-03	0.162198D-03	0.232260D
17.0	0.118926D-05	0.386760D-05	0.883710D-05	0.166415D-04	0.277449D-04	0.425244D-04	0.612660D
17.5	0.300394D-06	0.981787D-06	0.225424D-05	0.426530D-05	0.714440D-05	0.110003D-04	0.159191D
18.0	0.756493D-07	0.248126D-06	0.571698D-06	0.108542D-05	0.182420D-05	0.281800D-05	0.409126D
18.5	0.190105D-07	0.625111D-07	0.144387D-06	0.274803D-06	0.462953D-06	0.716857D-06	0.104317D
19.0	0.477010D-08	0.157133D-07	0.363586D-07	0.693192D-07	0.116980D-07	0.181443D-06	0.264474D
19.5	0.119562D-08	0.394355D-08	0.913630D-08	0.174403D-07	0.294673D-07	0.457607D-07	0.667800D
20.0	0.299456D-09	0.988591D-09	0.229237D-08	0.437974D-08	0.740648D-08	0.115116D-07	0.168138D

k/ξ	0.15	0.20	0.25	0.30	0.35	0.40	0.45
4.0	0.159246D 06	0.173715D 06	0.179473D 06	0.183155D 06	0.185357D 06	0.186817D 06	0.187790D
4.5	0.893588D 05	0.100094D 06	0.104836D 06	0.107792D 06	0.109604D 06	0.110788D 06	0.111591D
5.0	0.490769D 05	0.566832D 05	0.604791D 05	0.628097D 05	0.642678D 05	0.652223D 05	0.658775D
5.5	0.264267D 05	0.315462D 05	0.344463D 05	0.362363D 05	0.373820D 05	0.381408D 05	0.386679D
6.0	0.140027D 05	0.172640D 05	0.193514D 05	0.206786D 05	0.215535D 05	0.221441D 05	0.225605D
6.5	0.734888D 04	0.931121D 04	0.107169D 05	0.116587D 05	0.123035D 05	0.127503D 05	0.130717D
7.0	0.385593D 04	0.497129D 04	0.585574D 04	0.649057D 04	0.694545D 04	0.727128D 04	0.751232D
7.5	0.204407D 04	0.264519D 04	0.316685D 04	0.357087D 04	0.387564D 04	0.410313D 04	0.427758D
8.0	0.110447D 04	0.141447D 04	0.170490D 04	0.194705D 04	0.213979D 04	0.229107D 04	0.241212D
8.5	0.610907D 03	0.766344D 03	0.920533D 03	0.105745D 04	0.117230D 04	0.126802D 04	0.134803D
9.0	0.345368D 03	0.422934D 03	0.502085D 03	0.575616D 03	0.640517D 03	0.698288D 03	0.748335D
9.5	0.198225D 03	0.237830D 03	0.277863D 03	0.315910D 03	0.351293D 03	0.384659D 03	0.414088D
10.0	0.114348D 03	0.135536D 03	0.155992D 03	0.175460D 03	0.194516D 03	0.212941D 03	0.229210D
10.5	0.655100D 02	0.775167D 02	0.883780D 02	0.986107D 02	0.108900D 03	0.118638D 03	0.127263D
11.0	0.367897D 02	0.439558D 02	0.500764D 02	0.557646D 02	0.613396D 02	0.663782D 02	0.709744D
11.5	0.199776D 02	0.243875D 02	0.280464D 02	0.313725D 02	0.344257D 02	0.371156D 02	0.396871D
12.0	0.103469D 02	0.130584D 02	0.153126D 02	0.173032D 02	0.190328D 02	0.205741D 02	0.220725D
12.5	0.504618D 01	0.665631D 01	0.802806D 01	0.921767D 01	0.102380D 02	0.111622D 02	0.120308D
13.0	0.229261D 01	0.318883D 01	0.398343D 01	0.467751D 01	0.528308D 01	0.583033D 01	0.631710D
13.5	0.963141D 00	0.142066D 01	0.184781D 01	0.223243D 01	0.257685D 01	0.288517D 01	0.315015D
14.0	0.373043D 00	0.584624D 00	0.794271D 00	0.991462D 00	0.117319D 01	0.133649D 01	0.147802D
14.5	0.133465D 00	0.221803D 00	0.314952D 00	0.406937D 00	0.494598D 00	0.575401D 00	0.648607D
15.0	0.443841D-01	0.778221D-01	0.115259D 00	0.154079D 00	0.192545D 00	0.229517D 00	0.265059D
15.5	0.138471D-01	0.254337D-01	0.391252D-01	0.539908D-01	0.693469D-01	0.848351D-01	0.100608D
16.0	0.409613D-02	0.781873D-02	0.124248D-01	0.176350D-01	0.232384D-01	0.291582D-01	0.354884D
16.5	0.116107D-02	0.228534D-02	0.372963D-02	0.542114D-02	0.730677D-02	0.938053D-02	0.116858D
17.0	0.318378D-03	0.641730D-03	0.106959D-02	0.158503D-02	0.217711D-02	0.284988D-02	0.362034D
17.5	0.851370D-04	0.174712D-03	0.295951D-03	0.445280D-03	0.620964D-03	0.825731D-03	0.106554D
18.0	0.223448D-04	0.464694D-04	0.796809D-04	0.121285D-03	0.171141D-03	0.230381D-03	0.300917D
18.5	0.578469D-05	0.121481D-04	0.210188D-04	0.322725D-04	0.459465D-04	0.624280D-04	0.822915D
19.0	0.148273D-05	0.313591D-05	0.546161D-05	0.843965D-05	0.120954D-04	0.165481D-04	0.219618D
19.5	0.377336D-06	0.802127D-06	0.140368D-05	0.217920D-05	0.313832D-05	0.431544D-05	0.575572D
20.0	0.955351D-07	0.203827D-06	0.357911D-06	0.557528D-06	0.805738D-06	0.111203D-05	0.148850D

BICKLEY'S FUNCTION $Ki_3(x)$

$x = 0(0.01)10$

x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$
0.00	0.78539816	0.50	0.42635832	1.00	0.23784515	1.50	0.13464056
0.01	0.77547561	0.51	0.42132662	1.01	0.23512542	1.51	0.13313169
0.02	0.76570451	0.52	0.41635837	1.02	0.23243790	1.52	0.13164023
0.03	0.75608048	0.53	0.41145268	1.03	0.22978239	1.53	0.13016611
0.04	0.74659968	0.54	0.40660865	1.04	0.22715871	1.54	0.12870885
0.05	0.73725864	0.55	0.40182543	1.05	0.22456598	1.55	0.12726837
0.06	0.72805414	0.56	0.39710213	1.06	0.22200405	1.56	0.12584448
0.07	0.71898319	0.57	0.39243796	1.07	0.21947253	1.57	0.12443711
0.08	0.71004280	0.58	0.38783208	1.08	0.21697127	1.58	0.12304579
0.09	0.70123040	0.59	0.38328367	1.09	0.21449942	1.59	0.12167048
0.10	0.69254336	0.60	0.37879193	1.10	0.21205684	1.60	0.12031097
0.11	0.68397918	0.61	0.37435608	1.11	0.20964316	1.61	0.11896707
0.12	0.67553550	0.62	0.36997535	1.12	0.20725825	1.62	0.11763874
0.13	0.66721002	0.63	0.36564898	1.13	0.20490130	1.63	0.11632552
0.14	0.65900053	0.64	0.36137620	1.14	0.20257217	1.64	0.11502736
0.15	0.65090490	0.65	0.35715633	1.15	0.20027053	1.65	0.11374409
0.16	0.64292101	0.66	0.35298862	1.16	0.19799625	1.66	0.11247564
0.17	0.63504697	0.67	0.34887236	1.17	0.19574855	1.67	0.11122160
0.18	0.62728079	0.68	0.34480684	1.18	0.19352731	1.68	0.10998192
0.19	0.61962058	0.69	0.34079139	1.19	0.19133221	1.69	0.10875643
0.20	0.61206452	0.70	0.33682533	1.20	0.18916312	1.70	0.10754507
0.21	0.60461084	0.71	0.33290799	1.21	0.18701932	1.71	0.10634745
0.22	0.59725780	0.72	0.32903868	1.22	0.18490069	1.72	0.10516351
0.23	0.59000371	0.73	0.32521683	1.23	0.18280693	1.73	0.10399310
0.24	0.58284690	0.74	0.32144175	1.24	0.18073791	1.74	0.10283617
0.25	0.57578585	0.75	0.31771284	1.25	0.17869295	1.75	0.10169232
0.26	0.56881896	0.76	0.31402948	1.26	0.17667194	1.76	0.10056153
0.27	0.56194470	0.77	0.31039105	1.27	0.17467458	1.77	0.09944363
0.28	0.55516159	0.78	0.30679696	1.28	0.17270059	1.78	0.09833859
0.29	0.54846817	0.79	0.30324663	1.29	0.17074985	1.79	0.09724602
0.30	0.54186302	0.80	0.29973946	1.30	0.16882173	1.80	0.09616590
0.31	0.53534476	0.81	0.29627491	1.31	0.16691611	1.81	0.09509808
0.32	0.52891199	0.82	0.29285241	1.32	0.16503273	1.82	0.09404251
0.33	0.52256346	0.83	0.28947140	1.33	0.16317148	1.83	0.09299885
0.34	0.51629783	0.84	0.28613134	1.34	0.16133175	1.84	0.09196705
0.35	0.51011383	0.85	0.28283170	1.35	0.15951345	1.85	0.09094698
0.36	0.50401021	0.86	0.27957193	1.36	0.15771632	1.86	0.08993860
0.37	0.49798575	0.87	0.27635154	1.37	0.15594026	1.87	0.08894157
0.38	0.49203926	0.88	0.27316997	1.38	0.15418468	1.88	0.08795586
0.39	0.48616957	0.89	0.27002677	1.39	0.15244950	1.89	0.08698134
0.40	0.48037549	0.90	0.26692142	1.40	0.15073447	1.90	0.08601796
0.41	0.47465597	0.91	0.26385343	1.41	0.14903951	1.91	0.08506542
0.42	0.46900986	0.92	0.26082231	1.42	0.14736405	1.92	0.08412367
0.43	0.46343609	0.93	0.25782759	1.43	0.14570802	1.93	0.08319259
0.44	0.45793359	0.94	0.25486880	1.44	0.14407118	1.94	0.08227206
0.45	0.45250133	0.95	0.25194548	1.45	0.14245344	1.95	0.08136204
0.46	0.44713827	0.96	0.24905716	1.46	0.14085428	1.96	0.08046222
0.47	0.44184342	0.97	0.24620341	1.47	0.13927362	1.97	0.07957259
0.48	0.43661576	0.98	0.24338378	1.48	0.13771124	1.98	0.07869302
0.49	0.43145439	0.99	0.24059784	1.49	0.13616706	1.99	0.07782347

x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$
2.00	0.07696367	2.50	0.04430717	3.00	0.02564654	3.50	0.01490976
2.01	0.07611358	2.51	0.04382316	3.01	0.02536882	3.51	0.01474947
2.02	0.07527309	2.52	0.04334458	3.02	0.02509418	3.52	0.01459092
2.03	0.07444217	2.53	0.04287127	3.03	0.02482253	3.53	0.01443409
2.04	0.07362054	2.54	0.04240322	3.04	0.02455387	3.54	0.01427896
2.05	0.07280818	2.55	0.04194036	3.05	0.02428815	3.55	0.01412552
2.06	0.07200499	2.56	0.04148265	3.06	0.02402537	3.56	0.01397377
2.07	0.07121092	2.57	0.04103006	3.07	0.02376545	3.57	0.01382365
2.08	0.07042572	2.58	0.04058246	3.08	0.02350839	3.58	0.01367516
2.09	0.06964937	2.59	0.04013982	3.09	0.02325414	3.59	0.01352828
2.10	0.06888176	2.60	0.03970209	3.10	0.02300270	3.60	0.01338301
2.11	0.06812286	2.61	0.03926925	3.11	0.02275400	3.61	0.01323931
2.12	0.06737243	2.62	0.03884118	3.12	0.02250803	3.62	0.01309717
2.13	0.06663044	2.63	0.03841784	3.13	0.02226475	3.63	0.01295658
2.14	0.06589680	2.64	0.03799920	3.14	0.02202415	3.64	0.01281752
2.15	0.06517147	2.65	0.03758524	3.15	0.02178617	3.65	0.01267996
2.16	0.06445423	2.66	0.03717582	3.16	0.02155080	3.66	0.01254389
2.17	0.06374504	2.67	0.03677094	3.17	0.02131800	3.67	0.01240930
2.18	0.06304381	2.68	0.03637054	3.18	0.02108778	3.68	0.01227618
2.19	0.06235053	2.69	0.03597460	3.19	0.02086005	3.69	0.01214449
2.20	0.06166495	2.70	0.03558301	3.20	0.02063482	3.70	0.01201424
2.21	0.06098707	2.71	0.03519576	3.21	0.02041205	3.71	0.01188539
2.22	0.06031680	2.72	0.03481278	3.22	0.02019171	3.72	0.01175796
2.23	0.05965410	2.73	0.03443408	3.23	0.01997381	3.73	0.01163189
2.24	0.05899876	2.74	0.03405953	3.24	0.01975827	3.74	0.01150719
2.25	0.05835077	2.75	0.03368911	3.25	0.01954509	3.75	0.01138384
2.26	0.05771004	2.76	0.03332279	3.26	0.01933423	3.76	0.01126184
2.27	0.05707648	2.77	0.03296055	3.27	0.01912570	3.77	0.01114115
2.28	0.05645007	2.78	0.03260227	3.28	0.01891943	3.78	0.01102177
2.29	0.05583062	2.79	0.03224795	3.29	0.01871542	3.79	0.01090368
2.30	0.05521809	2.80	0.03189755	3.30	0.01851363	3.80	0.01078689
2.31	0.05461242	2.81	0.03155104	3.31	0.01831406	3.81	0.01067134
2.32	0.05401357	2.82	0.03120832	3.32	0.01811666	3.82	0.01055705
2.33	0.05342136	2.83	0.03086938	3.33	0.01792140	3.83	0.01044399
2.34	0.05283577	2.84	0.03053418	3.34	0.01772828	3.84	0.01033216
2.35	0.05225672	2.85	0.03020271	3.35	0.01753729	3.85	0.01022155
2.36	0.05168419	2.86	0.02987486	3.36	0.01734836	3.86	0.01011212
2.37	0.05111799	2.87	0.02955062	3.37	0.01716149	3.87	0.01000388
2.38	0.05055812	2.88	0.02922996	3.38	0.01697667	3.88	0.00989681
2.39	0.05000449	2.89	0.02891282	3.39	0.01679387	3.89	0.00979091
2.40	0.04945709	2.90	0.02859921	3.40	0.01661305	3.90	0.00968614
2.41	0.04891575	2.91	0.02828902	3.41	0.01643420	3.91	0.00958251
2.42	0.04838043	2.92	0.02798225	3.42	0.01625730	3.92	0.00947999
2.43	0.04785109	2.93	0.02767886	3.43	0.01608234	3.93	0.00937860
2.44	0.04732769	2.94	0.02737883	3.44	0.01590928	3.94	0.00927829
2.45	0.04681007	2.95	0.02708207	3.45	0.01573809	3.95	0.00917906
2.46	0.04629821	2.96	0.02678857	3.46	0.01556878	3.96	0.00908091
2.47	0.04579205	2.97	0.02649831	3.47	0.01540132	3.97	0.00898383
2.48	0.04529158	2.98	0.02621126	3.48	0.01523567	3.98	0.00888778
2.49	0.04479662	2.99	0.02592734	3.49	0.01507182	3.99	0.00879277

x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$
4.00	0.00869879	4.50	0.00509028	5.00	0.00298625	5.50	0.00175574
4.01	0.00860583	4.51	0.00503616	5.01	0.00295463	5.51	0.00173722
4.02	0.00851387	4.52	0.00498261	5.02	0.00292336	5.52	0.00171890
4.03	0.00842290	4.53	0.00492964	5.03	0.00289241	5.53	0.00170077
4.04	0.00833291	4.54	0.00487724	5.04	0.00286180	5.54	0.00168284
4.05	0.00824390	4.55	0.00482540	5.05	0.00283151	5.55	0.00166510
4.06	0.00815585	4.56	0.00477412	5.06	0.00280155	5.56	0.00164754
4.07	0.00806874	4.57	0.00472338	5.07	0.00277190	5.57	0.00163018
4.08	0.00798257	4.58	0.00467319	5.08	0.00274258	5.58	0.00161299
4.09	0.00789734	4.59	0.00462354	5.09	0.00271356	5.59	0.00159599
4.10	0.00781302	4.60	0.00457442	5.10	0.00268485	5.60	0.00157917
4.11	0.00772961	4.61	0.00452582	5.11	0.00265645	5.61	0.00156253
4.12	0.00764710	4.62	0.00447775	5.12	0.00262835	5.62	0.00154606
4.13	0.00756549	4.63	0.00443019	5.13	0.00260056	5.63	0.00152977
4.14	0.00748475	4.64	0.00438314	5.14	0.00257305	5.64	0.00151365
4.15	0.00740488	4.65	0.00433659	5.15	0.00254584	5.65	0.00149770
4.16	0.00732586	4.66	0.00429055	5.16	0.00251892	5.66	0.00148193
4.17	0.00724770	4.67	0.00424500	5.17	0.00249229	5.67	0.00146631
4.18	0.00717039	4.68	0.00419993	5.18	0.00246594	5.68	0.00145087
4.19	0.00709391	4.69	0.00415534	5.19	0.00243988	5.69	0.00143559
4.20	0.00701825	4.70	0.00411123	5.20	0.00241408	5.70	0.00142047
4.21	0.00694340	4.71	0.00406760	5.21	0.00238857	5.71	0.00140551
4.22	0.00686937	4.72	0.00402443	5.22	0.00236332	5.72	0.00139070
4.23	0.00679612	4.73	0.00398173	5.23	0.00233835	5.73	0.00137606
4.24	0.00672367	4.74	0.00393948	5.24	0.00231364	5.74	0.00136157
4.25	0.00665199	4.75	0.00389768	5.25	0.00228919	5.75	0.00134724
4.26	0.00658109	4.76	0.00385633	5.26	0.00226500	5.76	0.00133305
4.27	0.00651095	4.77	0.00381542	5.27	0.00224107	5.77	0.00131902
4.28	0.00644156	4.78	0.00377495	5.28	0.00221740	5.78	0.00130513
4.29	0.00637292	4.79	0.00373492	5.29	0.00219398	5.79	0.00129140
4.30	0.00630503	4.80	0.00369531	5.30	0.00217080	5.80	0.00127780
4.31	0.00623785	4.81	0.00365612	5.31	0.00214787	5.81	0.00126435
4.32	0.00617140	4.82	0.00361735	5.32	0.00212519	5.82	0.00125105
4.33	0.00610567	4.83	0.00357900	5.33	0.00210275	5.83	0.00123788
4.34	0.00604065	4.84	0.00354106	5.34	0.00208055	5.84	0.00122486
4.35	0.00597632	4.85	0.00350352	5.35	0.00205858	5.85	0.00121197
4.36	0.00591268	4.86	0.00346638	5.36	0.00203684	5.86	0.00119922
4.37	0.00584972	4.87	0.00342964	5.37	0.00201534	5.87	0.00118660
4.38	0.00578745	4.88	0.00339330	5.38	0.00199407	5.88	0.00117412
4.39	0.00572584	4.89	0.00335734	5.39	0.00197302	5.89	0.00116177
4.40	0.00566489	4.90	0.00332176	5.40	0.00195219	5.90	0.00114955
4.41	0.00560460	4.91	0.00328657	5.41	0.00193159	5.91	0.00113746
4.42	0.00554496	4.92	0.00325175	5.42	0.00191121	5.92	0.00112550
4.43	0.00548595	4.93	0.00321730	5.43	0.00189104	5.93	0.00111366
4.44	0.00542758	4.94	0.00318322	5.44	0.00187108	5.94	0.00110195
4.45	0.00536984	4.95	0.00314950	5.45	0.00185134	5.95	0.00109036
4.46	0.00531272	4.96	0.00311615	5.46	0.00183181	5.96	0.00107890
4.47	0.00525620	4.97	0.00308315	5.47	0.00181249	5.97	0.00106755
4.48	0.00520030	4.98	0.00305050	5.48	0.00179337	5.98	0.00105633
4.49	0.00514499	4.99	0.00301820	5.49	0.00177445	5.99	0.00104523

x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$
6.00	0.00103424	6.50	0.00061026	7.00	0.00036062	7.50	0.00021339
6.01	0.00102337	6.51	0.00060386	7.01	0.00035685	7.51	0.00021116
6.02	0.00101261	6.52	0.00059753	7.02	0.00035312	7.52	0.00020896
6.03	0.00100197	6.53	0.00059127	7.03	0.00034943	7.53	0.00020678
6.04	0.00099144	6.54	0.00058508	7.04	0.00034578	7.54	0.00020463
6.05	0.00098102	6.55	0.00057895	7.05	0.00034217	7.55	0.00020249
6.06	0.00097071	6.56	0.00057288	7.06	0.00033859	7.56	0.00020038
6.07	0.00096052	6.57	0.00056688	7.07	0.00033505	7.57	0.00019829
6.08	0.00095042	6.58	0.00056094	7.08	0.00033155	7.58	0.00019623
6.09	0.00094044	6.59	0.00055507	7.09	0.00032809	7.59	0.00019418
6.10	0.00093056	6.60	0.00054925	7.10	0.00032466	7.60	0.00019216
6.11	0.00092078	6.61	0.00054350	7.11	0.00032127	7.61	0.00019015
6.12	0.00091111	6.62	0.00053781	7.12	0.00031791	7.62	0.00018817
6.13	0.00090154	6.63	0.00053217	7.13	0.00031459	7.63	0.00018621
6.14	0.00089207	6.64	0.00052660	7.14	0.00031131	7.64	0.00018427
6.15	0.00088271	6.65	0.00052109	7.15	0.00030806	7.65	0.00018235
6.16	0.00087344	6.66	0.00051563	7.16	0.00030484	7.66	0.00018045
6.17	0.00086427	6.67	0.00051023	7.17	0.00030165	7.67	0.00017857
6.18	0.00085519	6.68	0.00050489	7.18	0.00029850	7.68	0.00017671
6.19	0.00084621	6.69	0.00049960	7.19	0.00029539	7.69	0.00017487
6.20	0.00083733	6.70	0.00049437	7.20	0.00029230	7.70	0.00017305
6.21	0.00082854	6.71	0.00048920	7.21	0.00028925	7.71	0.00017124
6.22	0.00081984	6.72	0.00048408	7.22	0.00028623	7.72	0.00016946
6.23	0.00081123	6.73	0.00047901	7.23	0.00028324	7.73	0.00016770
6.24	0.00080272	6.74	0.00047400	7.24	0.00028029	7.74	0.00016595
6.25	0.00079429	6.75	0.00046904	7.25	0.00027736	7.75	0.00016422
6.26	0.00078596	6.76	0.00046413	7.26	0.00027446	7.76	0.00016251
6.27	0.00077771	6.77	0.00045927	7.27	0.00027160	7.77	0.00016082
6.28	0.00076955	6.78	0.00045446	7.28	0.00026876	7.78	0.00015914
6.29	0.00076147	6.79	0.00044971	7.29	0.00026596	7.79	0.00015749
6.30	0.00075348	6.80	0.00044500	7.30	0.00026318	7.80	0.00015585
6.31	0.00074558	6.81	0.00044035	7.31	0.00026044	7.81	0.00015422
6.32	0.00073776	6.82	0.00043574	7.32	0.00025772	7.82	0.00015262
6.33	0.00073002	6.83	0.00043118	7.33	0.00025503	7.83	0.00015103
6.34	0.00072236	6.84	0.00042667	7.34	0.00025237	7.84	0.00014946
6.35	0.00071478	6.85	0.00042221	7.35	0.00024973	7.85	0.00014790
6.36	0.00070729	6.86	0.00041779	7.36	0.00024713	7.86	0.00014636
6.37	0.00069987	6.87	0.00041342	7.37	0.00024455	7.87	0.00014484
6.38	0.00069253	6.88	0.00040910	7.38	0.00024200	7.88	0.00014333
6.39	0.00068526	6.89	0.00040482	7.39	0.00023947	7.89	0.00014184
6.40	0.00067808	6.90	0.00040059	7.40	0.00023698	7.90	0.00014036
6.41	0.00067097	6.91	0.00039640	7.41	0.00023450	7.91	0.00013890
6.42	0.00066393	6.92	0.00039225	7.42	0.00023206	7.92	0.00013745
6.43	0.00065697	6.93	0.00038815	7.43	0.00022964	7.93	0.00013602
6.44	0.00065008	6.94	0.00038409	7.44	0.00022724	7.94	0.00013461
6.45	0.00064327	6.95	0.00038008	7.45	0.00022487	7.95	0.00013321
6.46	0.00063653	6.96	0.00037610	7.46	0.00022253	7.96	0.00013182
6.47	0.00062985	6.97	0.00037217	7.47	0.00022021	7.97	0.00013045
6.48	0.00062325	6.98	0.00036828	7.48	0.00021791	7.98	0.00012909
6.49	0.00061672	6.99	0.00036443	7.49	0.00021564	7.99	0.00012775

x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$	x	$Ki_3(x)$
8.00	0.00012642	8.50	0.00007498	9.00	0.00004451	9.50	0.00002645
8.01	0.00012510	8.51	0.00007420	9.01	0.00004405	9.51	0.00002617
8.02	0.00012380	8.52	0.00007343	9.02	0.00004359	9.52	0.00002590
8.03	0.00012251	8.53	0.00007266	9.03	0.00004314	9.53	0.00002564
8.04	0.00012124	8.54	0.00007191	9.04	0.00004269	9.54	0.00002537
8.05	0.00011998	8.55	0.00007116	9.05	0.00004225	9.55	0.00002511
8.06	0.00011873	8.56	0.00007042	9.06	0.00004181	9 56	0 00002485
8.07	0.00011749	8.57	0.00006969	9.07	0.00004138	9.57	0.00002459
8.08	0.00011627	8.58	0.00006897	9.08	0.00004095	9.58	0.00002434
8.09	0.00011506	8.59	0.00006825	9.09	0.00004053	9.59	0.00002408
8.10	0.00011387	8.60	0.00006755	9.10	0.00004011	9.60	0.00002384
8.11	0.00011268	8.61	0.00006684	9.11	0.00003969	9.61	0.00002359
8.12	0.00011151	8.62	0.00006615	9.12	0.00003928	9.62	0.00002334
8.13	0.00011035	8.63	0.00006546	9.13	0.00003887	9.63	0.00002310
8.14	0.00010920	8.64	0.00006478	9.14	0.00003847	9.64	0.00002286
8.15	0.00010807	8.65	0.00006411	9.15	0.00003807	9.65	0.00002263
8.16	0.00010694	8.66	0.00006345	9.16	0.00003768	9.66	0.00002239
8.17	0.00010583	8.67	0.00006279	9.17	0.00003729	9.67	0.00002216
8.18	0.00010473	8.68	0.00006214	9.18	0.00003690	9.68	0.00002193
8.19	0.00010364	8.69	0.00006149	9.19	0.00003652	9.69	0.00002171
8.20	0.00010256	8.70	0.00006085	9.20	0.00003614	9.70	0.00002148
8.21	0.00010150	8.71	0.00006022	9.21	0.00003577	9.71	0.00002126
8.22	0.00010044	8.72	0.00005960	9.22	0.00003540	9.72	0.00002104
8.23	0.00009940	8.73	0.00005898	9.23	0.00003503	9.73	0.00002082
8.24	0.00009837	8.74	0.00005837	9.24	0.00003467	9.74	0.00002061
8.25	0.00009734	8.75	0.00005776	9.25	0.00003431	9.75	0.00002039
8.26	0.00009633	8.76	0.00005716	9.26	0.00003395	9.76	0.00002018
8.27	0.00009533	8.77	0.00005657	9.27	0.00003360	9.77	0.00001997
8.28	0.00009434	8.78	0.00005598	9.28	0.00003325	9.78	0.00001977
8.29	0.00009336	8.79	0.00005540	9.29	0.00003291	9.79	0.00001956
8.30	0.00009239	8.80	0.00005483	9.30	0.00003257	9.80	0.00001936
8.31	0.00009143	8.81	0.00005426	9.31	0 00003223	9 81	0.00001916
8.32	0.00009048	8.82	0.00005370	9.32	0.00003190	9.82	0.00001896
8.33	0.00008954	8.83	0.00005314	9.33	0.00003157	9.83	0.00001877
8.34	0.00008861	8.84	0.00005259	9.34	0.00003124	9.84	0.00001857
8.35	0.00008769	8.85	0.00005204	9.35	0.00003092	9.85	0.00001838
8.36	0.00008678	8.86	0.00005150	9.36	0.00003060	9.86	0.00001819
8.37	0.00008588	8.87	0.00005097	9.37	0.00003028	9.87	0.00001800
8.38	0.00008498	8.88	0.00005044	9.38	0.00002996	9.88	0.00001782
8.39	0 00008410	8.89	0.00004992	9.39	0.00002965	9.89	0.00001763
8.40	0.00008323	8.90	0.00004940	9.40	0.00002935	9.90	0.00001745
8.41	0.00008236	8.91	0.00004889	9.41	0.00002904	9.91	0.00001727
8.42	0.00008151	8.92	0.00004838	9.42	0.00002874	9.92	0.00001709
8.43	0.00008066	8.93	0.00004788	9.43	0.00002845	9.93	0.00001691
8.44	0.00007982	8.94	0.00004738	9.44	0.00002815	9.94	0.00001674
8.45	0.00007899	8.95	0.00004689	9.45	0.00002786	9.95	0.00001657
8.46	0.00007817	8.96	0.00004640	9.46	0.00002757	9.96	0.00001640
8.47	0.00007736	8.97	0.00004592	9.47	0.00002729	9.97	0.00001623
8.48	0.00007656	8.98	0.00004545	9.48	0.00002700	9.98	0.00001606
8.49	0.00007576	8.99	0.00004498	9.49	0.00002672	9.99	0.00001589
						10.00	0.00001573