PUBLIKACIJE ELEKTROTEHNIČKOG FAKULTETA UNIVERZITETA U BEOGRADU
PUBLICATIONS DE LA FACULTÉ D'ÉLECTROTECHNIQUE DE L'UNIVERSITÉ À BELGRADE

SERIJA: MATEMATIKA I FIZIKA — SÉRIE: MATHÉMATIQUES ET PHYSIQUE

№ 230 — № 241 (1968)

236. INEQUALITIES FOR SIMPLEXES*

Žarko Živanović

Generalizations of theorems 6 and 6.1 published in [1] are given in this paper.

Theorem 1. *Let the simplex* S_{n+1} *is determined by the points* $A_1, \ldots, A_n, A_{n+1}$ *in n-dimensional Euclidean space, and let the simplex* S_i *is derived from the simplex* S_{n+1} *through the omission of point* A_i. *If* A_i^0 *is the point of intersection of the straight line* A_iM *and the simplex* S_i, *than the following inequality is valid*

$$\sum_{i=1}^{n+1} \frac{A_i A_i^0}{M A_i} \geqslant \frac{(n+1)^2}{n}. \tag{1}$$

The equality sign is valid if and only if M is the center of gravity of simplex S_{n+1}.

Proof. Let M_i represents the simplex which is derived from simplex S_{n+1} in such a way that the point A_i is substituted by the point M. If h_i and d_i are distances of points A_i and M from simplex S_i, then the following equality is valid

$$\frac{d_i}{h_i} = 1 - \frac{MA_i}{A_i A_i^0}. \tag{2}$$

If V and V_i are volumes of simplex S_{n+1} and M_i respectively, then the following equality is valid

$$\frac{V_i}{V} = \frac{d_i}{h_i}. \tag{3}$$

From (3) by adding in respect of i and using equality $\sum_{i=1}^{n+1} V_i = V$, we obtain the equality

$$\sum_{i=1}^{n+1} \frac{d_i}{h_i} = 1. \tag{4}$$

The following equality follows from (2) and (4)

$$\sum_{i=1}^{n+1} \frac{MA_i}{A_i A_i^0} = n. \tag{5}$$

* Received by Editors January 2, 1968 and presented Avril 20, 1968 by V. Volonec and Z. Kurnik.

The following inequality is valid for the arithmetic mean and harmonic mean ration for the same number of the same integers

(6) $$\frac{1}{n+1}\sum_{i=1}^{n+1}\frac{A_iA_i^0}{MA_i}\geqslant\frac{n+1}{\sum_{i=1}^{n+1}\frac{MA_i}{A_iA_i^0}}$$

From (5) and (6) we obtain the stated inequality which has to be proved.

Theorem 2. *The following inequality is valid in simplex* S_{n+1}

(7) $$\sum_{i=1}^{n+1}\frac{h_i}{d_i}\geqslant(n+1)^2.$$

Proof. From the inequality

$$\frac{1}{n+1}\sum_{i=1}^{n+1}\frac{h_i}{d_i}\geqslant\frac{n+1}{\sum_{i=1}^{n+1}\frac{d_i}{h_i}},$$

using (4), we arrive to (7).

The inequality (7) represents of the theorem 6.1 from [1].

REFERENCE

[1] Ž. Živanović, *Certaines inégalités relatives au triangle,* these Publications, № 181—№ 196 (1967), 69—72.