PUBLIKACIJE ELEKTROTEHNIČKOG FAKULTETA UNIVERZITETA U BEOGRADU
PUBLICATIONS DE LA FACULTÉ D'ÉLECTROTECHNIQUE DE L'UNIVERSITÉ À BELGRADE

SERIJA: MATEMATIKA I FIZIKA — SÉRIE: MATHÉMATIQUES ET PHYSIQUE

№ 166 (1966)

SOLUTION OF A PROBLEM PROPOSED BY D. S. MITRINOVIĆ*

Kovina Milošević-Rakočević

D. S. Mitrinović (Note N 2837, *The Mathematical Gazette*, 1959) has proposed the following problem:

Consider the arithmetical progressions

$$a_r,\ a_r+\alpha_r,\ \dots,\ a_r+n\alpha_r \qquad (r=1, 2, \dots, p);$$

$$b_s+n\beta_s,\ b_s+(n-1)\beta_s,\ \dots,\ b_s \qquad (s=1, 2, \dots, q),$$

and form the following expressions

$$(1) \qquad \prod_{r=1}^{p} a_r,\ \prod_{r=1}^{p}(a_r+\alpha_r),\ \dots,\ \prod_{r=1}^{p}(a_r+n\alpha_r);$$

$$(2) \qquad \prod_{s=1}^{q}(b_s+n\beta_s),\ \prod_{s=1}^{q}[b_s+(n-1)\beta_s],\ \dots,\ \prod_{s=1}^{q} b_s.$$

Find the sum of products of the corresponding elements of sequences (1) and (2).

We determine the proposed sum in the following way:

If we denote

$$h_1(n)=\prod_{r=1}^{p}(a_r+n\alpha_r)=\sum_{k=0}^{p} H_p^k n^k,$$

$$h_2(n)=\prod_{s=1}^{q}(b_s+n\beta_s)=\sum_{k=0}^{q} N_q^k n^k,$$

then the proposed sum is

$$f(n)=\sum_{k=0}^{n} h_1(k)\, h_2(n-k),$$

or

$$f(n)=\sum_{r=0}^{q} N_q^r \sum_{k=0}^{n}(n-k)^r h_1(k).$$

---

* Presented december 1, 1965 by D. S. Mitrinović.

This sum can be written in the developed form

$$f(n)=N_q^0 h_1(n)+g_0(n-1)(N_q^0+N_q^1 n+N_q^2 n^2+\cdots+N_q^q n^q)$$
$$-g_1(n-1)(N_q^1+2N_q^2 n+3N_q^3 n^2+\cdots+qN_q^q n^{q-1})$$
$$+g_2(n-1)\left(N_q^2+\binom{3}{2}N_q^3 n+\binom{4}{2}N_q^4 n^2+\cdots+\binom{q}{2}N_q^q n^{q-2}\right)$$
$$\vdots$$
$$+(-1)^q g_q(n-1)N_q^q,$$

or

$$f(n)=N_q^0 h_1(n)+\sum_{r=0}^{q}(-1)^r g_r(n-1)\frac{D^r h_2(n)}{r!}$$

where

$$g_r(n-1)=\sum_{k=0}^{n-1}k^r h_1(k).$$

Since the polynomial $g_r(n)$ is the solution of the difference equation

$$g_r(n)-g_r(n-1)=n^r\prod_{r=1}^{p}(a_r+n\alpha_r)=\sum_{k=0}^{p}H_p^k n^{k+r},$$

it follows that

$$g_r(n)=\sum_{k=0}^{p}H_p^k(k+r)!\ [B_{k+r+1}(n+1)-B_{k+r+1}(0)],$$

where $B_n(x)$ denotes Bernoulli's polynomial of degree n.

For the evaluation of the coefficients H_n^q, see:

Kovina Milošević-Rakočević: *Prilozi Teoriji i praksi Bernoulli-evih polinoma i brojeva.* Posebna izdanja Matematičkog instituta u Beogradu, t. 2 (1963) p. 118—119.