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DANCOFF'S CORRECTION IN THE CASE OF AN ARBITRARY ARRANGEMENT OF CYLINDRICAL RODS*

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SUMMARY

The equation of loci of points of the boundary of integration given in AE-16 (Stockholm, 1959) is generalized in the present work. Hence, the possibility of calculating Dancoff's correction factor for arbitrarily arranged cylindrical fuel rods either in a cluster or any tight lattice is offered. To illustrate the method the possibility of calculating Dan. coff's correction factor for the cluster of the Swedish power reactor Marviken is shown-

INTRODUCTION

In tight lattices in which the distances between the fuel elements are not much larger than the neutron mean free path in a moderator, the surface term in the usual formula for resonance integral, is reduced because of mutual geometrical shielding of the elements. This effect is still more important in the case of clustered fuel elements.

Dancoff's correction factor which expresses the relative reduction of the neutron current on one of the fuel elements due to the presence of the other fuel elements is [2]

$$C=\frac{J_s}{J_0}=\frac{2}{\sqrt{\pi}}\int Ki_3(\Sigma r)\cos\beta\,\mathrm{d}\beta \tag{1}$$

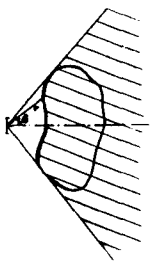

Fig. 1

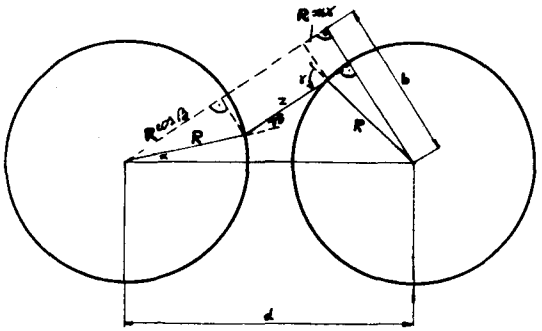

Fig. 2

* Presented December 15, 1965 by D. M. Ivanović.

J_s is that part of the total current which does not come to the fuel element from the shadowed part of the source (Fig. 1); J_0 is the current which would come to the fuel element if other elements were not present; $Ki_3(\Sigma r)$ is a special case of Bickley's functions defined earlier [4]. The integral is taken over the shadowed part of the source, Σ is the macroscopic scattering cross-section of the moderator. In deriving equation (1) it was assumed that

a) Fuel elements are suspended in the moderator of infinite extent.
b) Fuel element is absolutely black in the given small resonance energy range.
c) Only in one collision with the moderator atom is the neutron eliminated from this range.
d) The neutron source density over the whole moderator is homogeneous.

DANCOFF'S COORECTION FACTOR IN THE CASE OF ARBITRARILY ARRANGED FUEL RODS

In the case of two infinitely long cylindrical fuel elements (Fig. 2), as shown in [1], Dancoff's correction factor is given by the following expression

$$C=\frac{1}{\pi^2}\int_{-1}^{+1}\mathrm{d}u\int_{-1}^{+1}\mathrm{d}v\,\frac{Ki_3(z)}{\sqrt{\left(\frac{d}{R}\right)^2-(u+v)^2}} \tag{2}$$

where

$$u=\sin\beta,\quad v=\sin\gamma \tag{3}$$

Fig. 2 also shows the angles β, γ and the distance z between the points on the surfaces of the fuel elements in units of the neutron mean free path in a moderator. It also gives the meaning of d and R.

In case a third element (Fig. 3), or more of them, are positioned between the elements 1 and 2, then part of the integration domain in the plane (u, v) must be excluded, because part of the surface on the element 1 which

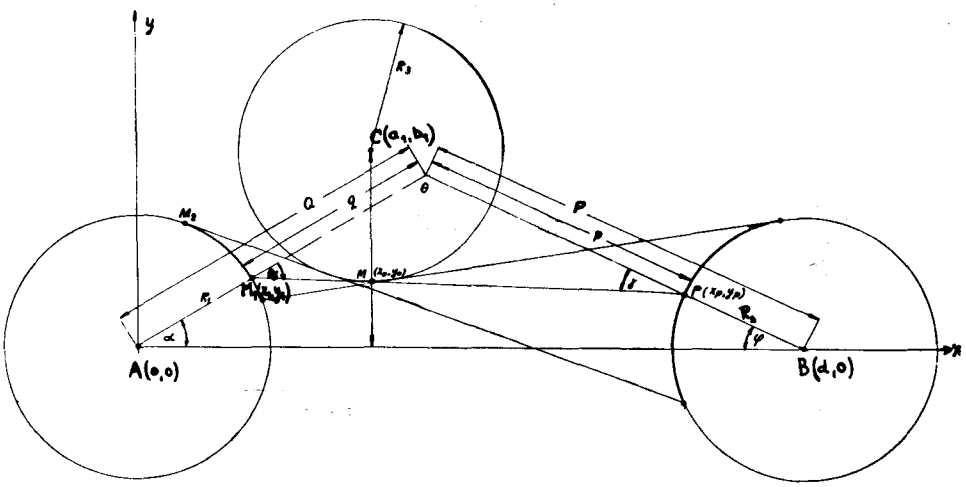

Fig. 3

is shadowed by element 2 is already shadowed by the closer element 3. It is necessary, therefore, to find the loci of points in the plane (u, v) by which part of the integration region is excluded. Ref. [1] discusses only the case of the symmetrical position of the element 3 and the equation of the loci is only a special case of the equation we give.

From Fig. 3, and from simple trigonometrical relations for coordinates of the point $M_1(x_1, y_1)$, when $R_1 = R_2 = R_3$, we write

$$y_1 = \frac{R}{d}\left\{-zu + R\left[v\sqrt{1-u^2} - u\sqrt{1-v^2}\right]\right\}$$

(4)

$$x_1 = \sqrt{R^2 - \left\{\frac{R}{d}\left[-zu + R\left(v\sqrt{1-u^2} - u\sqrt{1-v^2}\right)\right]\right\}^2}$$

If we draw a tangent from point $M_1(x_1, y_1)$ on circle 3, its direction coefficient is

$$i = \frac{R(x_1 - a_1) \mp (y_1 - b_1)S}{R(y_1 - b_1) \pm S(x_1 - a_1)} \tag{5}$$

and the y-intersection

$$j = \frac{R r^2}{R(y_1 - b_1) \pm S(x_1 - a_1)} + b_1 \tag{6}$$

The upper signs in (5) and (6) refer to the case $b_1 > 0$, the lower ones to $b_1 < 0$. Here

$$r^2 = (x_1 - a_1)^2 + (y_1 - b_1)^2, \qquad S = \sqrt{r^2 - R^2} \tag{7}$$

Intersection of this tangent with circle 2 determines the coordinates of point $P(x_p, y_p)$. The straight line through points B and P and the tangent determine the limiting value of the angle γ under which it is possible to see circle 2 from point M_1 (angles β and γ are measured from the normal on the circles at the points M_1 and M_p to the connecting line z; anti-clockwise direction is positive). Using the formula for sine of the angle between two straight lines, for loci of points, we write

$$v = \frac{j - i(d - a_1)}{R\sqrt{1 + i^2}} \tag{8}$$

This equation expresses the generalization of the case treated in [1], and is of considerable practical importance. It can be used to treat arbitrarily arranged fuel pins in a cluster or in a tight lattice.

Equation (8) was analyzed on a ZUSE-Z-23 digital computer. Results are shown in Fig. 4. In all cases the distance d between elements 1 and 2 was 5 cm and the radii of the rods $r = 1$ cm. The coordinates of points $C(a_1, b_1)$ are indicated on the corresponding curves. It is obvious that in the case of asymmetric position of element 3, the curve of the loci (8) is also asymmetric with respect to the straight line in the plane (u, v) drawn through points (0,0) and $(-1,1)$. In the case of symmetric position of element 3 the

loci (8) is also symmetric relative to the straight line. The latter case has been treated in [1] and it is abvious from Fig. 4 that it is only a special case of the loci (8).

To illustrate the method the possibility of calculating Dancoff's correction factor for the cluster of the Swedish power reactor Marviken, for which the cross section is given in Fig. 5, is shown. It can easily be shown that the

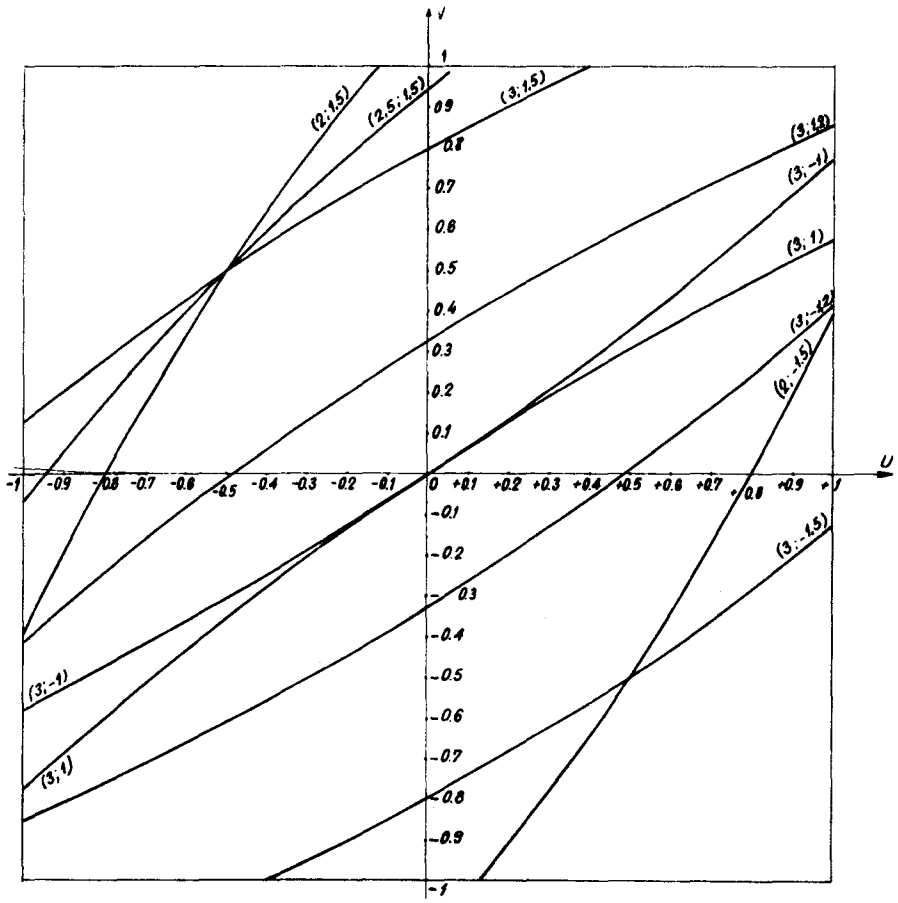

Fig. 4

effective surface in the case of the Marviken cluster is given by

$$S_{eff} = S_{tot}\left[1 - \frac{1}{6}\sum_{i,j} C_{i,j}\right] \tag{9}$$

where $C_{i,j}$ is the Dancoff's correction factor for the shadowing of rod i by rod j. Because of the reasons of symmetry it was only necessary to calculate 36

values $C_{i,j}$, given in the formulae (11). Each of these $C_{i,j}$ can be calculated using expression (3) with the fixed boundary or with the boundary given by equation (8).

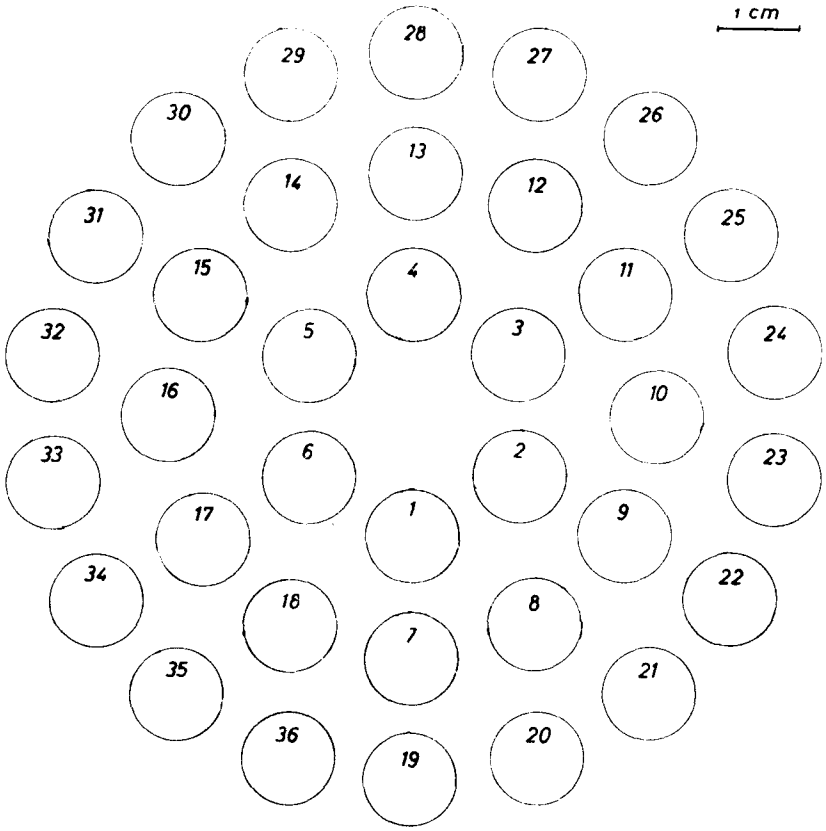

Fig. 5

The double sum in (9) can be expressed as

$$\sum_{i,j} C_{i,j} = \sum_j C_{1,j} + \sum_j C_{7,j} + \sum_j C_{8,j} + \sum_j C_{19,j} + \sum_j C_{20,j} + \sum_j C_{21,j} \tag{10}$$

where

$$\sum_j C_{1,j} = 3\,C_{1,7} + 2\,(C_{1,8} + C_{1,9} + C_{1,5} + C_{20,1} + C_{1,22} + C_{1,14}) + C_{1,4}$$

$$\sum_j C_{7,j} = 4\,C_{1,7} + 2\,(C_{7,20} + C_{7,21} + C_{7,6} + C_{7,9} + C_{7,10}) \tag{11}$$

$$\sum_j C_{8,j} = 2\,(C_{1,8} + C_{8,20} + C_{1,7} + C_{8,19} + C_{8,23} + C_{8,18} + C_{8,11} + C_{8,6} + C_{8,4} + C_{8,12}) + C_{8,14}$$

$$
\begin{aligned}
&\sum_j C_{19,j} = 3\,C_{1,7} + 2\,(C_{19,8} + C_{19,21} + C_{19,22} + C_{20,24} + C_{19,6}) \\
&\sum_j C_{20,j} = 2\,C_{1,7} + (C_{19,21} + C_{19,22} + C_{20,24}) + C_{20,7} \\
&\qquad + C_{20,8} + C_{20,9} + C_{20,1} + C_{20,18} \qquad (11) \\
&\sum_j C_{21,j} = 2\,(C_{1,7} + C_{19,21} + C_{21,24} + C_{21,35}) + C_{8,20} + C_{7,20} \\
&\qquad + C_{21,7} + C_{21,2} + C_{21,10}
\end{aligned}
$$

ACKNOWLEDGMENTS

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REMARK:

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