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AN INVESTIGATION OF THE FUNCTION $f(x)=x-(\sin x)^p(\cos x)^q$ IN THE INTERVAL $x\in(0,\ \pi/2)$ FOR DIFFERENT VALUES OF THE PARAMETERS $p\,(>1)$ AND $q\,(<0)$*

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This paper is in connexion with the preceding one in this *Publications* (D. S. Mitrinović — D. D. Adamović: *Sur une inégalité élémentaire où interviennent des fonctions trigonométriques*) and its aim is to complete and make the results concerning the case $p>1$, $q<0$ more precise.

An investigation of the function $f(x)$ for different values of the parameters p and q was made with a ZUSE-Z 23 digital computer.

It is evident that $f(0)=0$ and $f'(0)=1$, and that $f(x)\to-\infty$ when $x\to\pi/2$. In addition in the interval $x\in(0,\ \pi/2)$ the function $f(x)$ shifts over the positive values towards the negative ones. Consequently, the function $f(x)$ and its derivative function $f'(x)$ in the given interval have at least one zero.

The zeros for either pair of the parameters p and q were examined by iteration beginning from the upper limit of the interval. This makes it possible to avoid overrunning of the computer limits in case the values of argument x are close to the upper limits of the interval. If the condition of approaching zeros of the functions $f(x)$ and $f'(x)$ is assumed to be an approach over the negative values of the functions, the first zeros next to the upper limit of the interval will not be jumped over.

The investigation resulted in the following:

(1) For any pair of parameters p and q ($p=1$ to 3 with the step 0.25 and $q=-0.125$, -0.0625, -0.03125), the function $f(x)$ has only one zero in the interval $(0,\ \pi/2)$, which is in agreement with the corresponding result reported in the already cited paper by D. S. Mitrinović et D. D. Adamović. If either p or q increases, this zero approaches the upper limit of the interval. For example, for any value p and $q=-1/32$ zeros slightly differ from 89.9999698. (The value of the derivative function $f'(x)$ as a zero of the function increases as zero approaches the upper limit of the interval.)

(2) For some pairs p and q appear three zeros of the derivative function.

The problem arising thereof is:

Find the interval in the plane of parameters p and q in which occur the three extrema of the function $f(x)$ and examine how they behave as functions of p and q.

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* Presented by D. S. Mitrinović.

To solve this problem we must at least roughly know the interval in question. Varying the parameter p from 1 with the step 0.0625 and varying q from -0.015625 to -0.5 with the step -0.15625 for each value p, we shall determine the number of extrema of function $f(x)$ and their values in the interval $(0, \pi/2)$ for each pair of the parameters. The results of this examination are as follows:

(1) For some $p(>1)$ and $q(<0)$ the three extrema of the function $f(x)$ are in the interval $(0, \pi/2)$.

(2) With increasing p the three extrema of the function $f(x)$ occur for several q values (with the above step). The interval is limited at two sides with respect to q and it becomes larger as the parameter p increases.

(3) The values of the analogue extrema for the given p decrease with $q(<0)$ and more rapidly for smaller values of the parameter p.

(4) The values of the analogue extrema for the given q decrease with $p(>1)$ and the decrease is more rapid if the values q are smaller.

Now we are to determine the upper and the lower boundary curve of the interval of the three extreme values and to examin the minimum boundary value of the minimum, i. e. the minimum value of the pair of parameters near-by the critical point C of the boundary curve (Fig. 1).

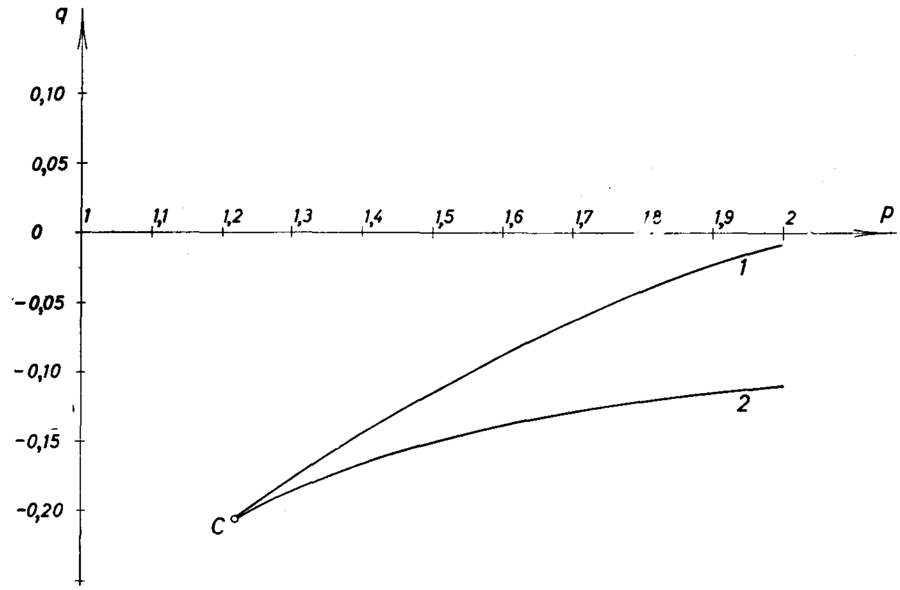

Fig. 1

For greater distances between curves 1 and 2 ($|\Delta q| > 1/64$) the curves were obtained by iteration with respect to q. When the distance becomes smaller than 1/64 a successive iteration of p and q is performed and the critical point C obtained

The entire logical development of the procedure is shown in the attached graph. The results are given in the table. The errors of the iteration of p and q are $\pm 2^{-20}$ and $\pm 2^{-22}$ respectively. Accordingly, the indefiniteness of the point C (Fig. 1) is $(2^{-20}\times 2^{-22})$.

Since the least minimum value of the function $f(x)$ exceeds zero, the function $f(x)$ for any p and q value in the interval (0, $\pi/2$ has only one zero.

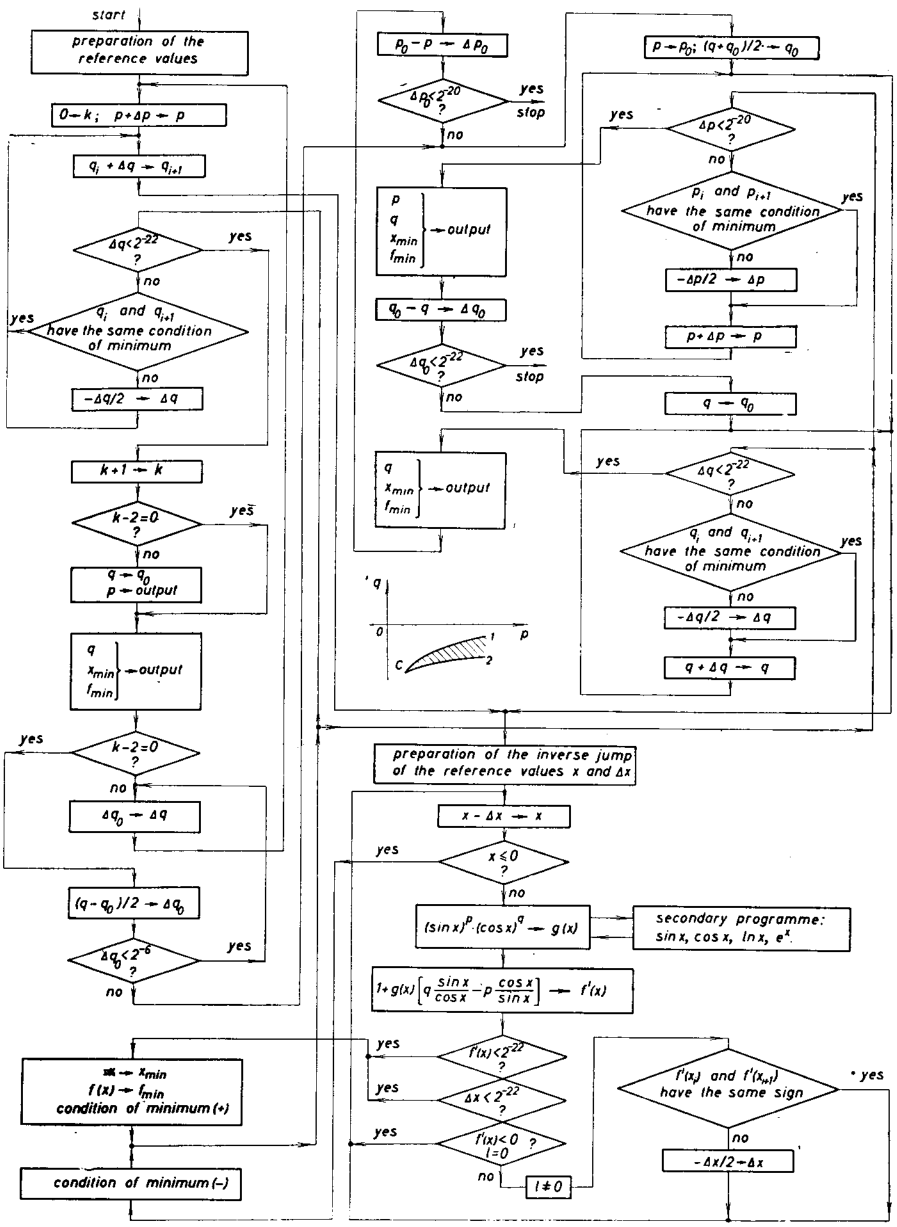

Fig. 2

THE BOUNDARY CURVE OF THE MINIMUM OF THE FUNCTION $f(x)=x-(\sin x)^p(\cos x)^q$ IN THE PLANE OF p AND q

p	q_1	min x_1	min f_1	q_2	min x_2	min f_2
1.875000	−0.024206	45.000000	0.258862	−0.113654	70.000000	0.216408
1.750000	−0.051539	45.000000	0.230317	−0.123597	70.000000	0.197694
1.625000	−0.082027	45.000000	0.199584	−0.134111	65.000000	0.177848
1.500000	−0.113311	40.000000	0.166982	−0.149140	63.127441	0.153386
1.375000	−0.149742	40.000000	0.131338	−0.167955	60.000000	0.125338
1.346542	−0.158849	40.000000	0.122769	−0.172794	60.000000	0.118447
1.295444	−0.175959	40.000000	0.106939	−0.183584	55.000000	0.104692
1.229657	−0.199423	40.000000	0.085681	−0.200141	50.000000	0.085463
1.2164060	−0.203933	45.000000	0.081341	−0.203934	50.000000	0.081353
1.2164057	−0.203933	45.000000	0.081341	−0.203934	50.000000	0.081353