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ON DIVERGENT SERIES OF POSITIVE TERMS*

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In their paper[1] Banerjee and Lahiri proved the following:

Theorem. *Let*

$$u(1)+u(2)+u(3)+\cdots \tag{1}$$

be a divergent series of positive terms for which

$$\lim_{n\to+\infty} u(n)=0. \tag{2}$$

Let P be a positive number. Then there is a subseries $u(n_1)+u(n_2)+u(n_3)+\cdots$ which converges to P.

Their proof is based on an effective construction of such a subseries. We give a simpler construction of another subseries with the same properties.

Proof. Let n_1 be the least integer such that $u(n_1)<P$. The existence of n_1 is a consequence of (2). Let n_2 be the least integer such that $n_2>n_1$ and $u(n_1)+u(n_2)<P$. Continuation of the construction yields the integers $n_1, n_2, n_3, \ldots$ such that

$$n_1<n_2<n_3<\cdots \tag{3}$$

$$S_k=\sum_{i=1}^{k} u(n_i)<P \qquad (k=1, 2, 3, \ldots). \tag{4}$$

We shall prove that

$$u(n_1)+u(n_2)+u(n_3)+\cdots=P. \tag{5}$$

From (4) we conclude that the series on the left-hand member of (5) is convergent and that its sum S (say) is not greater than P. Let us suppose that

$$u(n_1)+u(n_2)+u(n_3)+\cdots=S<P. \tag{6}$$

Taking (2) into account we conclude that there exists an integer m such that

$$u(n)<P-S \quad \text{for all} \quad n>n_m.$$

Since (1) is a divergent series we conclude that the sequence $\{n_i\}$ $(i=1, 2, 3, \ldots)$ does not contain all integers $>n_m$. Let r be the least integer such that $r>n_m$ and r is not contained in the sequence $\{n_i\}$ $(i=1, 2, 3, \ldots)$. Hence, $r-1=n_k$ for some k. We have

$$S_k+u(r)<S+(P-S)=P$$

which is in contradiction with the construction of the sequence $\{n_i\}$ $(i=1, 2, 3, \ldots)$. It folows that (6) is false. Therefore (5) is true. The theorem is established.

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* Presented December 20, 1964 by D. S. Mitrinović.

[1] C. R. Banerjee and B. K. Lahiri: *On subseries of divergent series*, Amer. Math. Monthly, 71 (1964), 767—768.