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CORRECT METHOD OF EVALUATION OF AVERAGE ENERGY OF NEUTRONS AFTER *n*-COLLISIONS

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1.1. The problem of the evaluation of average energy of neutrons after some number of collisions in the theory of slowing-down is very important. If neutrons with energy E_0 appear in the medium of mass number greater than unity, E_0 being much higher than the thermal agitation energy of mediums atoms, they will lose the energy in elastic collisions with them. The smallest value of the energy that may be reached by neutrons in one collision is αE_0, where $\alpha = [(A-1)/(A+1)]^2$. Let the frequency function for elastic collision be $h(E_0, E_1)$. The fraction of neutrons scattered into energy interval dE_1 around E_1, after first collision, will be $h(E_0, E_1)\, dE_1$. The average energy of neutrons after first collision is

$$\overline{E}_1 = \int\limits_{\alpha E_0}^{E_0} E_1\, h(E_0, E_1)\, dE_1.$$

The probability that neutrons will be scattered after second collision into dE_2 around E_2 is

$$h(E_0, E_1) \cdot h(E_1, E_2)\, dE_1\, dE_2.$$

The average energy of neutrons after second collision is

$$\overline{E}_2 = \int\limits_{\alpha E_0}^{E_0} \int\limits_{\alpha E_1}^{E_1} E_2\, h(E_0, E_1)\, h(E_1, E_2)\, dE_1\, dE_2.$$

Since the energy of neutrons after first collision (E_1) may have any value from the interval $(E_0, \alpha E_0)$, the integration over all values of E_1 must be caried out after integration over all possible values of E_2.

The probability that the neutrons will have the energy in dE_n around E_n after n-collisions is the product of frequency functions for elastic collision. Then

$$h(E_0, E_1)\; h(E_1, E_2)\; \ldots\; h(E_{n-1}, E_n)\, dE_1\, dE_2\; \ldots\; dE_m.$$

The average energy of neutrons after n-collisions will be

$$\bar{E}_n = \int_{\alpha E_0}^{E_0} \int_{\alpha E_1}^{E_1} \ldots \int_{\alpha E_{n-1}}^{E_{n-1}} E_n\, h(E_0, E_1)\, h(E_1, E_2) \ldots h(E_{n-1}, E_n)\, dE_1\, dE_2 \ldots dE_n.$$

1.2. In the case of isotropic scatering in the center of mass system the frequency function for elastic collision has the form

$$h(E_0, E_1) = \frac{1}{E_0(1-\alpha)}$$

and the probability that neutrons after n-collisions have the energy in interval dE_n around E_n is

$$\frac{1}{E_0(1-\alpha)} \frac{1}{E_1(1-\alpha)} \frac{1}{E_2(1-\alpha)} \cdots \frac{1}{E_{n-1}(1-\alpha)} dE_1\, dE_2 \ldots dE_n.$$

The average energy of neutrons after n-collisions is

$$\bar{E}_n = \int_{\alpha E_0}^{E_0} \int_{\alpha E_1}^{E_1} \int_{\alpha E_2}^{E_2} \int_{\alpha E_{n-1}}^{E_{n-1}} E_n \left[\frac{1}{E_0(1-\alpha)} \frac{1}{E_1(1-\alpha)} \cdots \frac{1}{E_{n-1}(1-\alpha)}\right] dE_1\, dE_2 \cdots dE_n.$$

The integration being completed, we finde

$$\bar{E}_n = \left(\frac{1+\alpha}{2}\right)^n E_0.$$

The same result can be achieved if all neutrons after $(n-1)$ collisions are assumed to have energy equal to the average energy after $(n-1)$ collisions. Method for evaluation of average energy of neutrons after n-collisions, often given in literature, is based on this assumption. Such model is not adequate to the real situation, and the concordance of the results come from the assumption that the scattering in the center-of-mass system is isotropic, i. e. because of the nature of chosen type of frequency function for elastic collision. If the analytic expression for this function is of any other kind, for evaluation of average energy of neutrons after n-collisions, the method given in section 1.1 must be followed.

REFERENCE

[1] R. V. Meghreblian, D. K. Holmes (Oak Ridge National Laboratory): *Reactor Analysis*, Mc-Graw-Hill Book Company, inc. N. Y., L., 1960.