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A REMARK CONCERNING THE LOWER BOUND OF

$$\frac{x_1}{x_2+x_3}+\frac{x_2}{x_3+x_4}+\cdots+\frac{x_n}{x_1+x_2}$$

Bogdan Bajšanski

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Let

$$f_n(x_1, x_2, \ldots, x_n)=\frac{x_1}{x_2+x_3}+\frac{x_2}{x_3+x_4}+\cdots+\frac{x_{n-2}}{x_{n-1}+x_n}+\frac{x_{n-1}}{x_n+x_1}+\frac{x_n}{x_1+x_2},$$

$$i_n=\inf_{x_i>0} f_n(x_1, x_2, \ldots, x_n).$$

Evidently, $i_n \leqslant \frac{n}{2}$. The problem which has received some attention in the last time is to find for what values of n

(1) $$i_n=\frac{n}{2}.$$

In this note we prove the following

Theorem. If n is an odd number >1, and if $i_n<\frac{n}{2}$, then $i_{n+1}<\frac{n+1}{2}$.

Our proof depends on the following simple

Lemma. If x_i, $i=1, 2, \ldots, 2k-1$, are real numbers, then exist at least one i such that $x_i \in [x_{i-1}, x_{i+1}]$, $(x_0=x_{2k-1},\ x_{2k}=x_1)$.

Proof of the lemma. Suppose the contrary and write

$$p_i=(x_i-x_{i-1})(x_i-x_{i+1}).$$

Then $p_i>0$ for $i=1, 2, \ldots, 2k-1$. So $\prod_{i=1}^{2k-1} p_i>0$. But

$$\prod_{i=1}^{2k-1} p_i=(-1)^{2k-1}\prod_{i=1}^{2k-1}(x_i-x_{i-1})^2 \leqslant 0.$$

Proof of the theorem. There are $n=2k-1$ numbers $x_i>0$ such that $f_{2k-1}(x_1, x_2, \ldots, x_{2k-1})<\frac{2k-1}{2}$. According to the lemma, let

$$(x_i-x_{i-1})(x_i-x_{i+1}) \leqslant 0 \text{ for } i=j.$$

Then

(2) $$f_{2k}(x_1, x_2, \ldots, x_{j-1}, x_j, x_j, x_{j+1}, \ldots, x_{2k-1})$$
$$= f_{2k-1}(x_1, x_2, \ldots, x_{2k-1}) + \frac{x_{j-1}}{2x_j} + \frac{x_j}{x_j + x_{j+1}} - \frac{x_{j-1}}{x_j + x_{j+1}}$$
$$< \frac{2k-1}{2} + \frac{2x_j(x_j - x_{j-1}) + x_{j-1}(x_j + x_{j+1})}{2x_j(x_j + x_{j+1})}.$$

But

$$2x_j(x_j - x_{j-1}) + x_{j-1}(x_j + x_{j+1})$$
$$= (x_j - x_{j-1})(x_j - x_{j+1}) + x_j(x_j + x_{j+1}) < x_j(x_j + x_{j+1}),$$

and so the lefthand side in (2) is less than

$$\frac{2k-1}{2} + \frac{1}{2}.$$

Hence

$$i_{n+1} < \frac{n+1}{2}.$$

Remark. From the theorem just proved and from the result of D. Đoković (see *Proceedings of the Glasgow Mathematical Association*, 1963) that (1) is true for $n = 8$, it follows in particular that (1) ist true for $n = 7$, a result which was not known before.

See also: D. S. Mitrinović: *Zbornik matematičkih problema*, t. 1, Belgrade 1962, p. 356—359.

Izdavač
ELEKTROTEHNIČKI FAKULTET
Beograd — Bulevar Revolucije 73

Tehnički urednik i korektor
DRAGOSLAV ALEKSIĆ

Slagač
MIODRAG ĐUKIĆ

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