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712. A GENERALIZATION OF A LINEAR FUNCTIONAL EQUATION, II*

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The functional equation to be considered is a certain generalization of the functional equation

$$(0)\qquad (-1)^n f(x_1, \ldots, x_n) - f(x_2, \ldots, x_{n+1}) + \sum_{i=1}^{n} (-1)^{i+1} f(x_1, x_2, \ldots, x_i + x_{i+1}, \ldots, x_{n+1}) = 0$$

to the case where all the figuring functions are different:

$$(1)\qquad (-1)^n F_{n+1}(x_1, \ldots, x_n) - F_{n+2}(x_2, \ldots, x_{n+1}) + \sum_{i=1}^{n} (-1)^{i+1} F_i(x_1, \ldots, x_i + x_{i+1}, \ldots, x_{n+1}) = 0.$$

D. S. MITRINOVIĆ and D. Ž. DJOKOVIĆ [**1**] have obtained the general differentiable solution of equation (0). In certain particular cases (1) has been solved by the author in [**2**], and the general solution of equation (1) was also obtained but without proof.

In this paper we deduce the general differentiable solution of equation (1). The form of general solution given here is simplier than that of paper [**2**].

We prove the following

Theorem. *The general differentiable solution of equation* (1) *is*

$$(2)\qquad F_1(x_1, \ldots, x_r) = A_{11}(x_1 + x_2, x_3, \ldots, x_n) - A_{12}(x_1, x_2 + x_3, x_4, \ldots, x_n) + \cdots + (-1)^n A_{1,n-1}(x_1, \ldots, x_{n-2}, x_{n-1} + x_n) + (-1)^{n+1} A_{1n}(x_1, \ldots, x_{n-1}) - C_1(x_2, \ldots, x_n),$$

* Presented April 25, 1980 by D. Ž. DJOKOVIĆ.

$$
\begin{aligned}
F_i(x_1, \ldots, x_n) = {} & A_{1,i-1}(x_1+x_2, x_3, \ldots, x_n) - A_{2,i-1}(x_1, x_2+x_3, x_4, \ldots, x_n) \\
& + \cdots + (-1)^i A_{i-1,i-1}(x_1, \ldots, x_{i-1}+x_i, \ldots, x_n) \\
& + (-1)^{i+1} A_{ii}(x_1, \ldots, x_i+x_{i+1}, \ldots, x_n) \\
& + (-1)^{i+2} A_{i,i+1}(x_1, \ldots, x_{i+1}+x_{i+2}, \ldots, x_n) + \cdots \\
& + (-1)^n A_{i,n-1}(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) \\
& + (-1)^{n+1} A_{in}(x_1, \ldots, x_{n-1}) - C_i(x_2, \ldots, x_n) \\
& (i=2, \ldots, n), \\
F_{n+1}(x_1, \ldots, x_n) = {} & A_{1n}(x_1+x_2, x_3, \ldots, x_n) - A_{2n}(x_1, x_2+x_3, x_4, \ldots, x_n) \\
& + \cdots + (-1)^n A_{n-1,n}(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) \\
& + (-1)^{n+1} A_{nn}(x_1, \ldots, x_{n-1}) - C_{n+1}(x_2, \ldots, x_n), \\
F_{n+2}(x_1, \ldots, x_n) = {} & C_2(x_1+x_2, x_3, \ldots, x_n) - C_3(x_1, x_2+x_3, x_4, \ldots, x_n) \\
& + \cdots + (-1)^n C_n(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) \\
& + (-1)^{n+1} C_{n+1}(x_1, \ldots, x_{n-1}) - C_1(x_2, \ldots, x_n),
\end{aligned}
$$

where $A_{11}, \ldots, A_{nn}, C_1, \ldots, C_n$ are arbitrary differentiable functions.

Proof. In order to prove that (1) implies (2), first, let us differentiate (1) with respect to x_1 and substitute $x_1=0$ into the result, then using the notations

$$\frac{\partial}{\partial x_1} F_i(x_1, \ldots, x_n)\,|_{x_1=0} = a_{1,i-1}(x_2, x_3, \ldots, x_n) \qquad (i=2, \ldots, n+1),$$

we obtain immediately

$$
\begin{aligned}
(3) \quad \frac{\partial}{\partial x_2} F_1(x_2, \ldots, x_{n+1}) = {} & a_{11}(x_2+x_3, x_4, \ldots, x_{n+1}) - a_{12}(x_2, x_3+x_4, \ldots, x_{n+1}) \\
& + \cdots + (-1)^n a_{1,n-1}(x_2, \ldots, x_{n-1}, x_n+x_{n+1}) + (-1)^{n+1} a_{1n}(x_2, \ldots, x_n).
\end{aligned}
$$

Integrating this with respect to x_2, one obtain

$$
\begin{aligned}
(4) \quad F_1(x_2, \ldots, x_{n+1}) = {} & A_{11}(x_2+x_3, x_4, \ldots, x_{n+1}) - A_{12}(x_2, x_3+x_4, \ldots, x_{n+1}) \\
& + \cdots + (-1)^n A_{1,n-1}(x_2, \ldots, x_{n-1}, x_n+x_{n+1}) \\
& + (-1)^{n+1} A_{1n}(x_2, \ldots, x_n) - C_1(x_3, \ldots, x_{n+1}),
\end{aligned}
$$

where $\frac{\partial}{\partial x_2} A_{1i}(x_2, \ldots, x_n) = a_{1i}(x_2, \ldots, x_n)$ $(i=1, \ldots, n)$, $C_1(x_3, \ldots, x_{n+1})$

is an arbitrary differentiable function.

Putting this into (1) we have

$$(5)\quad A_{11}(x_1+x_2+x_3, x_4, \dots, x_{n+1})-A_{12}(x_1+x_2, x_3+x_4, x_5, \dots, x_{n+1})$$
$$+\cdots+(-1)^n A_{1,n-1}(x_1+x_2, x_3, \dots, x_{n-1}, x_n+x_{n+1})$$
$$+(-1)^{n+1} A_{1n}(x_1+x_2, x_3, \dots, x_n)-C_1(x_2, x_3, \dots, x_n)$$
$$+(-1)^n F_{n+1}(x_1, x_2, \dots, x_n)-F_{n+2}(x_2, x_3, \dots, x_{n+1})$$
$$+\sum_{i=2}^{n}(-1)^{i+1} F_i(x_1, \dots, x_{i-1}, x_i+x_{i+1}, \dots, x_{n+1})=0.$$

By putting $x_2=0$ into (5) and replacing x_{i+1} by x_i ($i=2, 3, \dots, n$) we have

$$(6)\quad F_2(x_1\ x_2, \dots, x_n)=A_{11}(x_1+x_2, x_3, \dots, x_n)-A_{22}(x_1, x_2+x_3, \dots, x_n)$$
$$+\cdots+(-1)^n A_{2,n-1}(x_1, x_2, \dots, x_{n-2}, x_{n-1}+x_n)$$
$$+(-1)^{n+1} A_{2n}(x_1, x_2, \dots, x_{n-1})-C_2(x_2, x_3, \dots, x_n),$$

where

$$A_{2i}(x_1, x_2, \dots, x_{n-1})=A_{1i}(x_1, x_2, \dots, x_{n-1})-F_{i+1}(x_1, 0, x_2, \dots, x_{n-1})$$

($i=2, \dots, n$),

$$C_2(x_2, x_3, \dots, x_n)=C_1(x_2, x_3, \dots, x_n)+F_{n+2}(0, x_2, x_3, \dots, x_n).$$

Substituting (6) into (5) we obtain

$$(7)\quad -A_{12}(x_1+x_2, x_3+x_4, x_5, \dots, x_{n+1})+A_{13}(x_1+x_2, x_3, x_4+x_5, \dots, x_{n+1})$$
$$-\cdots+(-1)^n A_{1,n-1}(x_1+x_2, x_3, \dots, x_{n-1}, x_n+x_{n+1})$$
$$+(-1)^{n+1} A_{1n}(x_1+x_2, x_3, \dots, x_n)+A_{22}(x_1, x_2+x_3+x_4, x_5, \dots, x_{n+1})$$
$$-A_{23}(x_1, x_2+x_3, x_4+x_5, \dots, x_{n+1})+\cdots$$
$$+(-1)^{n+1} A_{2,n-1}(x_1, x_2+x_3, \dots, x_{n-1}, x_n+x_{n+1})$$
$$+(-1)^{n+2} A_{2n}(x_1, x_2+x_3, x_4, \dots, x_n)-C_1(x_3, x_4, \dots, x_{n+1})$$
$$+C_2(x_2+x_3, x_4, \dots, x_{n+1})+(-1)^n F_{n+1}(x_1, \dots, x_n)$$
$$-F_{n+2}(x_2, \dots, x_{n+1})+\sum_{i=3}^{n}(-1)^{i+1} F_i(x_1, \dots, x_{i-1}, x_i+x_{i+1}, \dots, x_{n+1})=0.$$

We will show by induction that the first k $(2 \leq k \leq n)$ unknown functions $F_j (j=1, 2, \ldots, k)$ which satisfy equation (1) may be written as (4) and the following form

$$(8) \quad \begin{aligned} F_j(x_1, \ldots, x_n) &= A_{1,j-1}(x_1+x_2, x_3, \ldots, x_n) - A_{2,j-1}(x_1, x_2+x_3, \ldots, x_n) \\ &+ \cdots + (-1)^j A_{j-1,j-1}(x_1, \ldots, x_{j-1}+x_j, \ldots, x_n) \\ &+ (-1)^{j+1} A_{jj}(x_1, \ldots, x_j+x_{j+1}, \ldots, x_n) \\ &+ (-1)^{j+2} A_{j,j+1}(x_1, \ldots, x_{j+1}+x_{j+2}, \ldots, x_n) \\ &+ \cdots + (-1)^n A_{j,n-1}(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) \\ &+ (-1)^{n+1} A_{jn}(x_1, x_2, \ldots, x_{n-1}) - C_j(x_2, x_3, \ldots, x_n) \quad (j=2, \ldots, k), \end{aligned}$$

and finally the remaining $n-k+2$ unknown functions will satisfy the following equation

$$(9) \quad \begin{aligned} &(-1)^{k+1} A_{1k}(x_1+x_2, x_3, \ldots, x_{k+1}+x_{k+2}, \ldots, x_{n+1}) \\ &+ (-1)^{k+2} A_{1,k+1}(x_1+x_2, x_3, \ldots x_{k+2}+x_{k+3}, \ldots, x_{n+1}) \\ &+ \cdots + (-1)^n A_{1,n-1}(x_1+x_2, x_3, \ldots, x_n+x_{n+1}) \\ &+ (-1)^{n+1} A_{1n}(x_1+x_2, x_3, \ldots, x_n) \\ &- [(-1)^{k+1} A_{2k}(x_1, x_2+x_3, x_4, \ldots, x_{k+1}+x_{k+2}, \ldots, x_{n+1}) \\ &+ (-1)^{k+2} A_{2,k+1}(x_1, x_2+x_3, x_4, \ldots, x_{k+2}+x_{k+3}, \ldots, x_{n+1}) + \cdots \\ &+ (-1)^n A_{2,n-1}(x_1, x_2+x_3, x_4, \ldots, x_n+x_{n+1}) + (-1)^{n+1} A_{2n}(x_1, x_2+x_3, x_4, \ldots, x_n)] \\ &+ \cdots + (-1)^{k-1} [(-1)^{k+1} A_{kk}(x_1, \ldots, x_k+x_{k+1}+x_{k+2}, \ldots, x_{n+1}) \\ &+ (-1)^{k+2} A_{k,k+1}(x_1, \ldots, x_k+x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_{n+1}) + \cdots \\ &+ (-1)^n A_{k,n-1}(x_1, \ldots, x_k+x_{k+1}, x_{k+2}, \ldots, x_n+x_{n+1}) \\ &+ (-1)^{n+1} A_{kn}(x_1, \ldots, x_k+x_{k+1}, \ldots, x_n)] \\ &- C_1(x_3, x_4, \ldots, x_{n+1}) + C_2(x_2+x_3, x_4, \ldots, x_{n+1}) \\ &- \cdots + (-1)^k C_k(x_2, \ldots, x_k+x_{k+1}, \ldots, x_{n+1}) \\ &+ (-1)^n F_{n+1}(x_1, x_2, \ldots, x_n) - F_{n+2}(x_2, x_3, \ldots, x_{n+1}) \\ &+ \sum_{i=k+1}^{n} (-1)^{i+1} F_i(x_1, \ldots, x_i+x_{i+1}, \ldots, x_{n+1}) = 0. \end{aligned}$$

REMARK. For $k=n$ we define $\sum_{n+1}^{n} = 0$.

In fact, it follows from (4), (6) and (7) that the statement is valid for $k=2$. Suppose that the statement is valid for k where $2 \leqq k \leqq n-1$. Setting $x_{k+1}=0$ and replacing x_{i+1} by x_i ($i=k+1,\ k+2,\ \ldots,\ n$) in (9) we get

$$
\begin{aligned}
(10)\quad F_{k+1}(x_1, x_2, \ldots, x_n) &= A_{1k}(x_1+x_2, x_3, \ldots, x_n) - A_{2k}(x_1, x_2+x_3, \ldots, x_n) \\
&+ \cdots + (-1)^{k+1} A_{kk}(x_1, \ldots, x_{k-1}, x_k+x_{k+1}, \ldots, x_n) \\
&+ [F_{k+2}(x_1, \ldots, x_k, 0, x_{k+1}+x_{k+2}, \ldots, x_n) \\
&- A_{1,k+1}(x_1+x_2, x_3, \ldots, x_k, 0, x_{k+1}+x_{k+2}, \ldots, x_n) \\
&+ A_{2,k+1}(x_1, x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}+x_{k+2}, \ldots, x_n) - \cdots \\
&+ (-1)^{k-1} A_{k-1,k+1}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}+x_{k+2}, x_{k+3}, \ldots, x_n) \\
&+ (-1)^k A_{k,k+1}(x_1, \ldots, x_k, x_{k+1}+x_{k+2}, \ldots, x_n)] \\
&- [F_{k+3}(x_1, \ldots, x_k, 0, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n) \\
&- A_{1,k+2}(x_1+x_2, x_3, \ldots, x_k, 0, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n) \\
&+ A_{2,k+2}(x_1, x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n) - \cdots \\
&+ (-1)^{k-1} A_{k-1,k+2}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n) \\
&+ (-1)^k A_{k,k+2}(x_1, \ldots, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n)] + \cdots \\
&+ (-1)^{n-k-2} [F_n(x_1, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-2}, x_{n-1}+x_n) \\
&- A_{1,n-1}(x_1+x_2, x_3, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-2}, x_{n-1}+x_n) \\
&+ A_{2,n-1}(x_1, x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-2}, x_{n-1}+x_n) - \cdots \\
&+ (-1)^{k-1} A_{k-1,n-1}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, \ldots, x_{n-2}, x_{n-1}+x_n) \\
&+ (-1)^k A_{k,n-1}(x_1, \ldots, x_{n-2}, x_{n-1}+x_n)] \\
&+ (-1)^{n-k-1} [F_{n+1}(x_1, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-1}) \\
&- A_{1n}(x_1+x_2, x_3, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-1}) \\
&+ A_{2n}(x_1, x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-1}) - \cdots \\
&+ (-1)^{k-1} A_{k-1,n}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, \ldots, x_{n-1}) \\
&+ (-1)^k A_{kn}(x_1, x_2, \ldots, x_{n-1})] \\
&+ (-1)^k [F_{n+2}(x_2, \ldots, x_k, 0, x_{k+1}, \ldots, x_n) \\
&+ C_1(x_3, \ldots, x_k, 0, x_{k+1}, \ldots, x_n) \\
&- C_2(x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, \ldots, x_n) + \cdots \\
&+ (-1)^{k-2} C_{k-1}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, \ldots, x_n) \\
&+ (-1)^{k-1} C_k(x_2, x_3, \ldots, x_n)].
\end{aligned}
$$

Now we define

$$(11)\quad (-1)^{k+2}A_{k+1,\,k+i}(x_1, x_2, \ldots, x_{n-1}) = F_{k+i+1}(x_1, \ldots, x_k, 0, x_{k+1}, \ldots x_{n-1})$$
$$-A_{1,\,k+i}(x_1+x_2, x_3, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-1})$$
$$+A_{2,\,k+i}(x_1, x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, \ldots, x_{n-1}) - \cdots$$
$$+(-1)^{k-1}A_{k-1,\,k+i}(x_1, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, \ldots, x_{n-1})$$
$$+(-1)^{k}A_{k,\,k+i}(x_1, x_2, \ldots, x_{n-1}) \quad (i=1, \ldots, n-k),$$

$$(12)\quad C_{k+1}(x_2, x_3, \ldots, x_n) = (-1)^{k-1}[F_{n+2}(x_2, \ldots, x_k, 0, x_{k+1}, \ldots, x_n)$$
$$+C_1(x_3, \ldots, x_k, 0, x_{k+1}, \ldots, x_n)$$
$$-C_2(x_2+x_3, x_4, \ldots, x_k, 0, x_{k+1}, \ldots, x_n) + \cdots$$
$$+(-1)^{k-2}C_{k-1}(x_2, \ldots, x_{k-2}, x_{k-1}+x_k, 0, x_{k+1}, \ldots, x_n)$$
$$+(-1)^{k-1}C_k(x_2, x_3, \ldots, x_n)].$$

(10) with (11) and (12) finally yield

$$(13)\quad F_{k+1}(x_1, x_2, \ldots, x_n) = A_{1k}(x_1+x_2, x_3, \ldots, x_n) - A_{2k}(x_1, x_2+x_3, x_4, \ldots, x_n)$$
$$+\cdots+(-1)^{k+1}A_{kk}(x_1, \ldots, x_{k-1}, x_k+x_{k+1}, \ldots, x_n)$$
$$+(-1)^{k+2}A_{k+1,\,k+1}(x_1, \ldots, x_k, x_{k+1}+x_{k+2}, \ldots, x_n)$$
$$+(-1)^{k+3}A_{k+1,\,k+2}(x_1, \ldots, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_n) + \cdots$$
$$+(-1)^{n}A_{k+1,\,n-1}(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) + (-1)^{n+1}A_{k+1,\,n}(x_1, x_2, \ldots, x_{n-1})$$
$$-C_{k+1}(x_2, x_3, \ldots, x_n).$$

Substituting (13) into (9) and eliminating $A_{1k}, A_{2k}, \ldots, A_{kk}$, we obtain

$$(14)\quad (-1)^{k+2}A_{1,\,k+1}(x_1+x_2, x_3, \ldots, x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_{n+1}) + \cdots$$
$$+(-1)^{n}A_{1,\,n-1}(x_1+x_2, x_3, \ldots, x_n+x_{n+1}) + (-1)^{n+1}A_{1n}(x_1+x_2, x_3, \ldots, x_n)$$
$$-[(-1)^{k+2}A_{2,\,k+1}(x_1, x_2+x_3, x_4, \ldots, x_{k+2}+x_{k+3}, \ldots, x_{n+1}) + \cdots$$
$$+(-1)^{n}A_{2,\,n-1}(x_1, x_2+x_3, x_4, \ldots, x_n+x_{n+1})$$
$$+(-1)^{n+1}A_{2n}(x_1, x_2+x_3, x_4, \ldots, x_n)] + \cdots$$
$$+(-1)^{k+1}[(-1)^{k+2}A_{k,\,k+1}(x_1, \ldots, x_{k-1}, x_k+x_{k+1}, x_{k+2}+x_{k+3}, \ldots, x_{n+1})$$
$$+\cdots+(-1)^{n}A_{k,\,n-1}(x_1, \ldots, x_{k-1}, x_k+x_{k+1}, x_{k+2}, \ldots, x_n+x_{n+1})$$
$$+(-1)^{n+1}A_{kn}(x_1, \ldots, x_{k-1}, x_k+x_{k+1}, \ldots, x_n)]$$
$$+(-1)^{k}[(-1)^{k+2}A_{k+1,\,k+1}(x_1, \ldots, x_k, x_{k+1}+x_{k+2}+x_{k+3}, \ldots, x_{n+1}) + \cdots$$

$$+(-1)^n A_{k+1,n-1}(x_1, \dots, x_k, x_{k+1}+x_{k+2}, x_{k+3}, \dots, x_n+x_{n+1})$$
$$+(-1)^{n+1} A_{k+1,n}(x_1, \dots, x_k, x_{k+1}+x_{k+2}, \dots, x_n)]$$
$$-C_1(x_3, x_4, \dots, x_{n+1})+C_2(x_2+x_3, x_4, \dots, x_{n+1})-\cdots$$
$$+(-1)^k C_k(x_2, \dots, x_{k-1}, x_k+x_{k+1}, \dots, x_{n+1})$$
$$+(-1)^{k+1} C_{k+1}(x_2, \dots, x_k, x_{k+1}+x_{k+2}, \dots, x_{n+1})$$
$$+(-1)^n F_{n+1}(x_1, x_2, \dots, x_n)-F_{n+2}(x_2, x_3, \dots, x_{n+1})$$
$$+\sum_{i=k+2}^{n}(-1)^{i+1} F_i(x_1, \dots, x_{i-1}, x_i+x_{i+1}, \dots, x_{n+1})=0.$$

It follows from (13) and (14) that the statement remains valid for $k+1$. Thus (8) and (9) hold for $2 \leqq k \leqq n$.

By setting $k=n$, equation (9) becomes

(15) $$(-1)^{n+1} A_{1n}(x_1+x_2, x_3, \dots, x_n)+(-1)^{n+2} A_{2n}(x_1, x_2+x_3, x_4, \dots, x_n)+\cdots$$
$$+(-1)^{2n-1} A_{n-1,n}(x_1, \dots, x_{n-2}, x_{n-1}+x_n)+(-1)^{2n} A_{nn}(x_1, x_2, \dots, x_{n-1})$$
$$-C_1(x_3, x_4, \dots, x_{n+1})+C_2(x_2+x_3, x_4, \dots, x_{n+1})-\cdots$$
$$+(-1)^n C_n(x_2, \dots, x_{n-1}, x_n+x_{n+1})+(-1)^n F_{n+1}(x_1, x_2, \dots, x_n)$$
$$-F_{n+2}(x_2, x_3, \dots, x_{n+1})=0.$$

Setting $x_{n+1}=0$ in (15) we obtain

(16) $$F_{n+1}(x_1, x_2, \dots, x_n)=A_{1n}(x_1+x_2, x_3, \dots, x_n)-A_{2n}(x_1, x_2+x_3, x_4, \dots, x_n)$$
$$+\cdots+(-1)^n A_{n-1,n}(x_1, \dots, x_{n-2}, x_{n-1}+x_n)$$
$$+(-1)^{n+1} A_{nn}(x_1, x_2, \dots, x_{n-1})-C_{n+1}(x_2, x_3, \dots, x_n),$$

where

$$C_{n+1}(x_2\ x_3, \dots, x_n)=(-1)^{n+1}[C_1(x_3, x_4, \dots, x_n, 0)$$
$$-C_2(x_2+x_3, x_4, \dots, x_n, 0)$$
$$+\cdots+(-1)^n C_{n-1}(x_2, \dots, x_{n-2}, x_{n-1}+x_n, 0)$$
$$+(-1)^{n+1} C_n(x_2, x_3, \dots, x_n)+F_{n+2}(x_2, x_3, \dots, x_n, 0)].$$

Substituting (16) into (15) we obtain

$$-C_1(x_3, x_4, \dots, x_{n+1})+C_2(x_2+x_3, x_4, \dots, x_{n+1})$$
$$-\cdots+(-1)^n C_n(x_2, \dots, x_{n-1}, x_n+x_{n+1})$$
$$+(-1)^{n+1} C_{n+1}(x_2, x_3, \dots, x_n)-F_{n+2}(x_2, x_3, \dots, x_{n+1})=0,$$

i.e.

$$(17)\quad F_{n+2}(x_1, x_2, \ldots, x_n) = C_2(x_1+x_2, x_3, \ldots, x_n) - C_3(x_1, x_2+x_3, x_4, \ldots, x_n) + \cdots + (-1)^n C_n(x_1, \ldots, x_{n-2}, x_{n-1}+x_n) + (-1)^{n+1} C_{n+1}(x_1, x_2, \ldots, x_{n-1}) - C_1(x_2, x_3, \ldots, x_n).$$

From (4), (8) (taking $k=n$), (16) and (17) the theorem is proved.

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GENERALIZACIJA LINEARNE FUNKCIONALNE JEDNAČINE, II

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U radu je dato opšte diferencijabilno rešenje funkcionalne jednačine (1), koja predstavlja generalizaciju jednačina iz radova [1] i [2].