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# 710. A NOTE ON A PAPER OF H. GUPTA CONCERNING POWERS OF TWO AND THREE*

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The following question of P. ERDÖS was considered in [**1**]: does there exist an integer $m \neq 0, 2, 8$ such that $2^m$ is a sum of distinct povers of 3? It was shown there that there is no such integer in the interval (8, 4734). The aim of this note is to observe that a simple counting argument shows that there cannot be very many such integers $m$. More precisely we prove the following evaluation:

**Theorem.** *If $N(T)$ denotes the number of nonnegative integers $m \leqq T$ with the property that $2^m$ can be written as a sum of distinct powers of 3, then*

$$N(T) \leqq 1.62\, T^{\log 2/\log 3}$$

*and in particular the density of the set of such $m$' s is zero.*

***Proof.*** We only push further the analysis in the proof of theorem 4 of [**1**]. Observe first that if we have

$$2^m = 3^{m_0} + 3^{m_1} + \cdots + 3^{m_s} \tag{1}$$

with $m_0 < m_1 < \cdots < m_s$ then necessarily $m_0 = 0$. Now let $k$ be any positive integer and reduce the equality (1) (mod $3^k$). Then the terms with $m_j \geqq k$ will vanish and the right-hand side of (1) will take one of the values

$$1 + \varepsilon_1 \cdot 3 + \varepsilon_2 \cdot 3^2 + \cdots + \varepsilon_{k-1} \cdot 3^{k-1}$$

where each $\varepsilon_i$ equals 0 or 1. Thus we have $2^{k-1}$ possibilities for the value of $2^m$ (mod $3^k$) and as 2 is a primitive root for every power of 3 we see that there are only $2^{k-1}$ residue classes $r_1, \ldots, r_{2^{k-1}}$ (mod $2 \cdot 3^{k-1}$) in which $m$ can lie. As the number of nonnegative integers lying in an arithmetical pro-

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gression of difference $d$ and not exceeding $T$ is at most equal to $x/d+1$ we obtain that for any fixed $k$ the inequality

$$N(T)\leqq\sum_{j=1}^{2^{k-1}}\#\{n\leqq T: n\equiv r_j\ (\mathrm{mod}\ 2\cdot 3^{k-1})\}\leqq\frac{2^{k-1}}{2\cdot 3^{k-1}}\,T+2^{k-1}$$

holds. Choosing now $k=\left[\frac{\log(3/2)+\log T}{\log 3}\right]$ we obtain our assertion.

## REFERENCE

1. H. Gupta: *Powers of 2 and sums of distinct powers of 3*. These Publications № **602**—№ **633** (1978), 151—158.

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## JEDNA NOTA O ČLANKU H. GUPTA

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U radu je dokazana teorema koja glasi:

Ako $N(T)$ označava broj nenegativnih brojeva $m\leqq T$ sa osobinom da se $2^m$ može napisati kao zbir različitih potencija broja 3, tada je $N(T)\leqq 1.62\ T^{\log 2/\log 3}$.