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583. A FEW REMARKS ON A PREVIOUS PAPER REGARDING THE CONVERGENCE OF CERTAIN SEQUENCES*

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In this note we give three remarks which supplement and correct the result of paper [**1**].

1. Using the concepts defined in [**1**] we can formulate the following:

Theorem. *Let E be a complete metric space and let sphere $S(z, r) \subset E$. Furthermore let $f: E^p \to E$ such that*

$$d\big(f(u_1, \dots, u_p), f(u_2, \dots, u_{p+1})\big) \leq A\big(d(u_1, u_2), \dots, d(u_p, u_{p+1})\big)$$

holds for every $u_i \in S(z, r)$ $(i = 1, \dots, p+1)$.

Let $A: [\theta, h]^p \to G$ $([\theta, h] \subset G)$ be nondecreasing and continuous with respect to sequences, such that the equation

$$y = A(y, \theta, \dots, \theta) + A(\theta, y, \dots, \theta) + \cdots + A(\theta, \dots, \theta, y) \qquad (y \in [\theta, h])$$

has the unique solution θ. Also, let there exists $q \in [\theta, h]$ such that $d\big(z, f(z, \dots, z)\big) \leq q$, $q + A(r, \theta, \dots, \theta) + A(\theta, r, \dots, \theta) + \cdots + A(\theta, \dots, \theta, r) \leq r$ where $A(b, \dots, b) \leq b$ and $A(b, \theta, \dots, \theta) + A(\theta, b, \dots, \theta) + \cdots + A(\theta, \dots, \theta, b) \leq b$ for $2r \leq b$ and let the series $\sum_{n=0}^{+\infty} A_n$ converge, where

$$A_0 = b, \; A_k = A(A_{k-1}, \dots, A_{k-1}) \qquad (k = 1, 2, \dots).$$

Then, the sequence (x_n) defined by $x_{n+p} = f(x_n, \dots, x_{n+p-1})$, where $x_1, \dots, x_p \in S(z, r)$ are arbitrary chosen

1° *converges in $S(z, r)$;*

2° *the unique solution of the equation $x = f(x, \dots, x)$ is $x = \lim_{n \to +\infty} x_n$.*

The proof of the above theorem is similar to the proof of theorem from [**1**], and we shall therefore omit it.

2. The theorem given in [**1**] should be modified in such a way that the condition (2) is replaced by

$$d\big(f(u_1, \dots, u_p)\, f(v_1, \dots, v_p)\big) \leq A\big(d(u_1, v_1), \dots, d(u_p, v_p)\big)$$

for every $u_i, v_i \in S(z, r)$ $(i = 1, \dots, p)$.

3. Corollaries 2 and 3 given in [**1**] are, in fact corollaries to the theorem given above, and not to the (modified) theorem from [**1**].

REFERENCE

1. V. Lj. KOCIĆ: *A theorem on the convergence of sequences defined by recurrent relations*. These Publications № **498** — № **541** (1975), 149 — 152.

* Presented April 2, 1977 by J. D. KEČKIĆ.